Fundamentals of DSP Part 1

Introduction to Digital Signal Processing

Fundamentals of Digital Signal Processing
Content

Part 1: **What is a signal?**

Time and frequency domain

- **Fourier transformation**

Part 2: Digitizing signals

- Sampling
- Aliasing

Part 3: Effects to be aware of when converting to digital

- Quantization
- Leakage

Counter-measures to ensure your digital data is valid

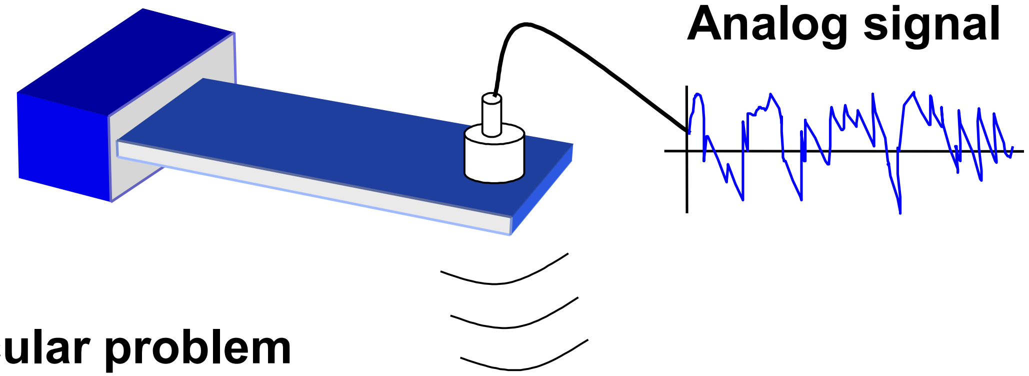
Signals

Signal: measurable quantity carrying information about some physical phenomenon

- Pressure, displacement, acceleration, …
- Temperature, voltage, biomedical potential (EKG, EEG, ...)

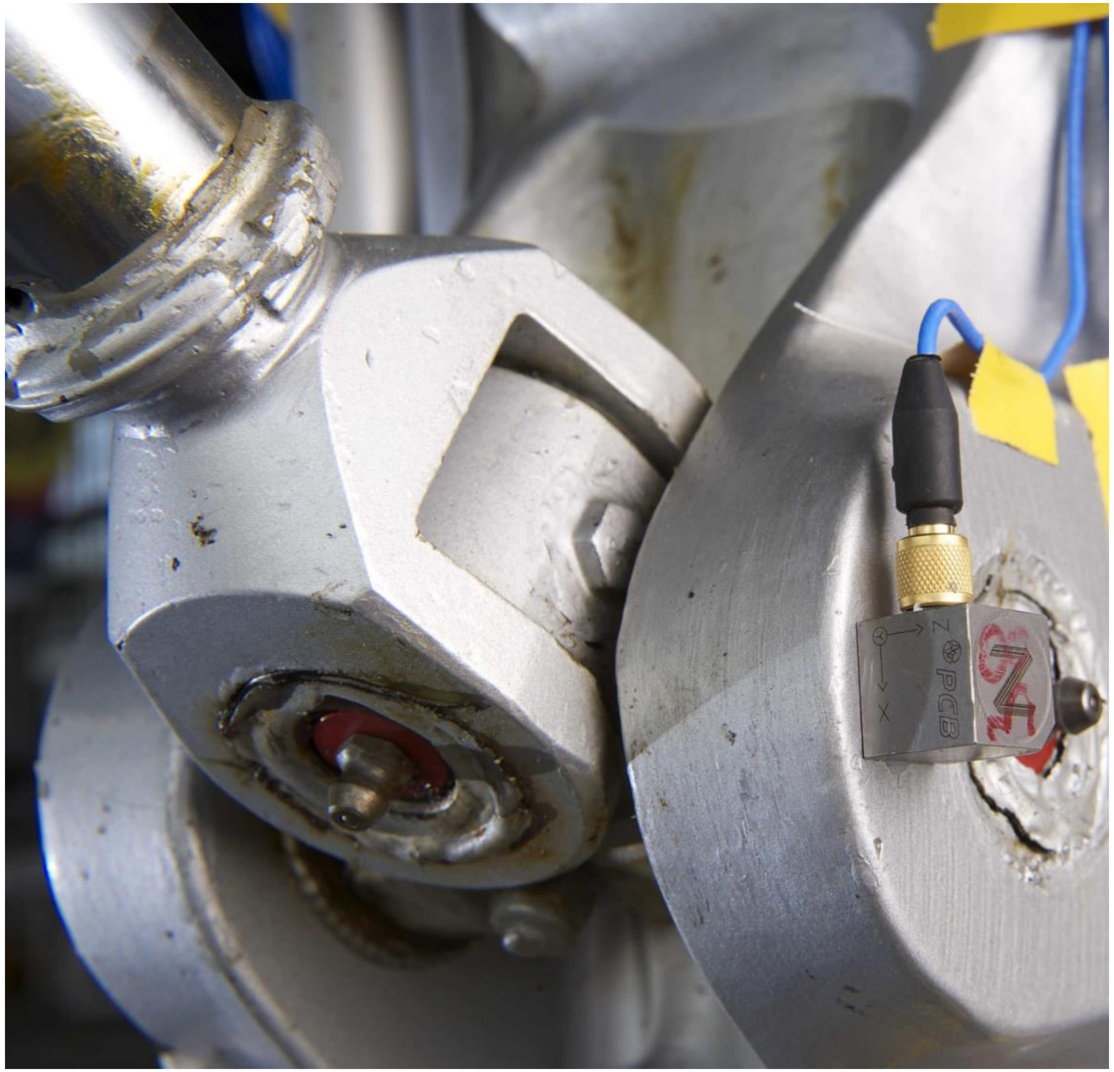
The signal is generated by a structure and detected by a sensor or transducer

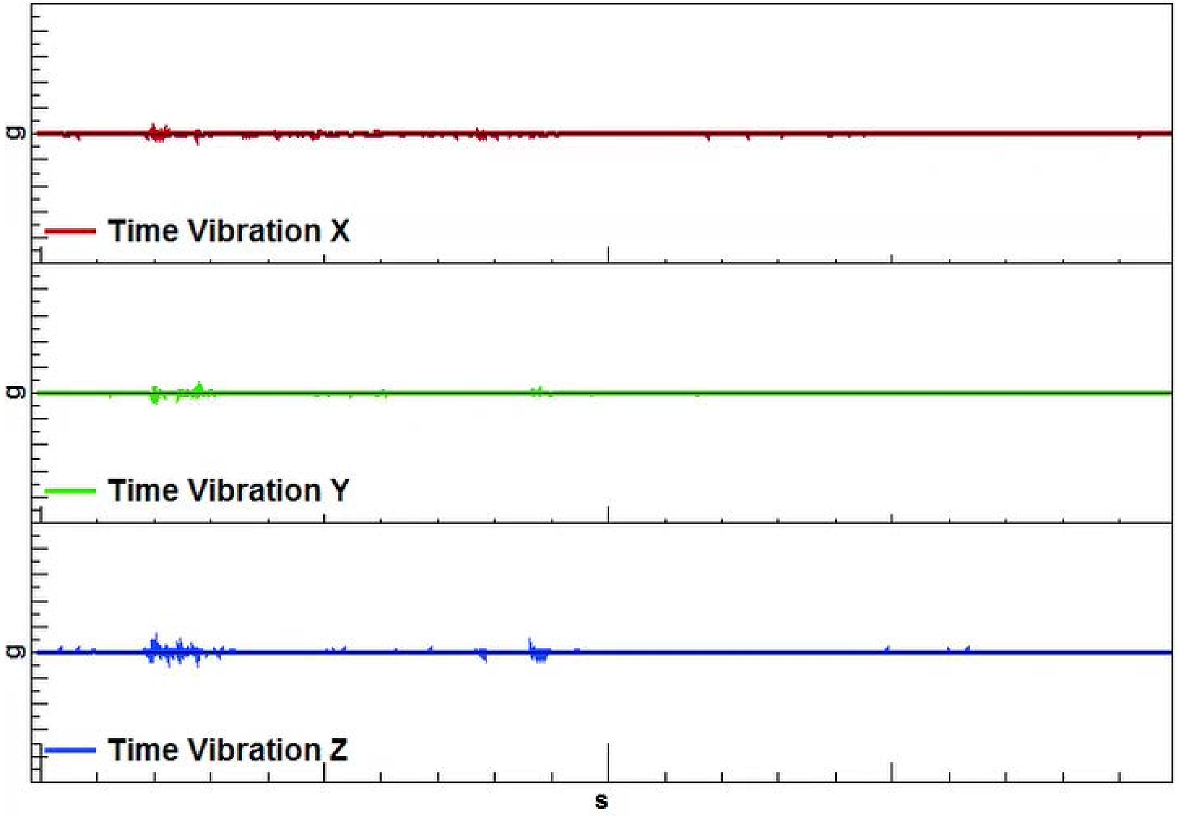
- Accelerometer: acceleration → voltage
- Microphone: pressure → voltage
- Strain Gauge: strain (deformation) → voltage
- Thermocouple: temperature changes → voltage



The signal is what you want to analyse in view of a particular problem

Signals

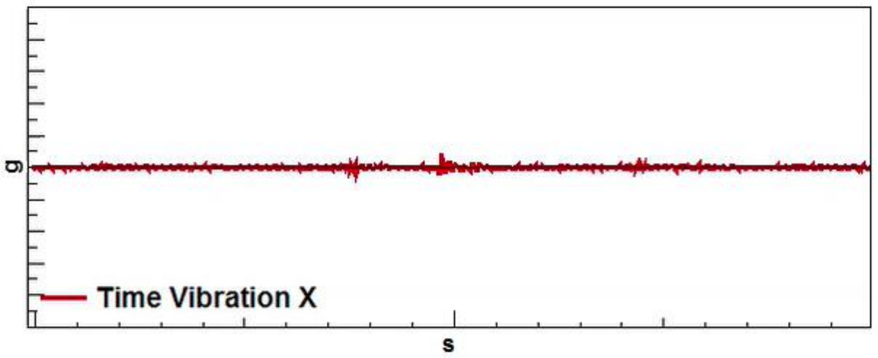




Signal Processing

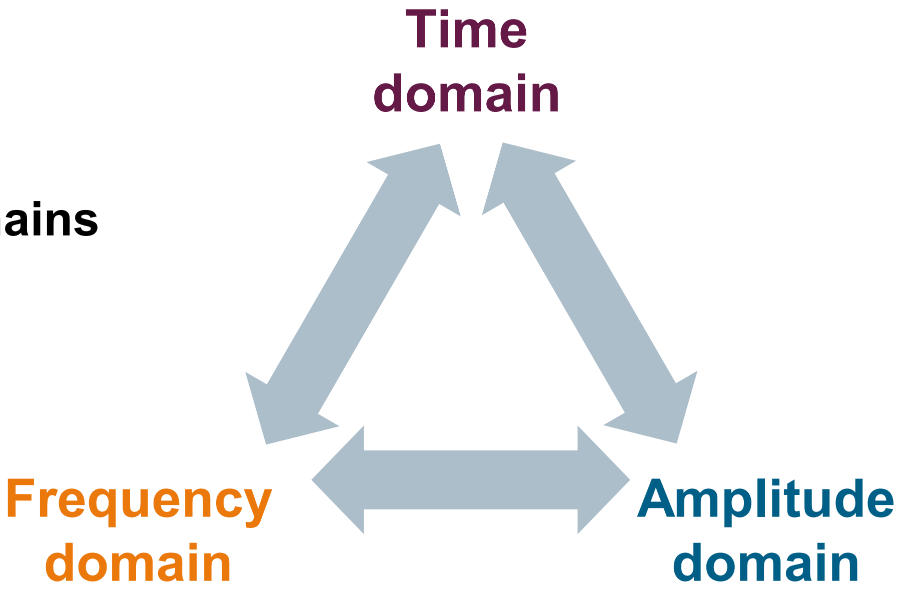
Signal processing: specific manipulations of the measured signal to

- Extract key information
- Understand physics
- Provide input data for specific analysis
- Confront simulation results with reality
- Modify the signal for specific applications



Signal processing transforms the signal to different domains

- Time domain
- Frequency domain
- Amplitude domain
- Laplace domain
- …



Time, frequency and amplitude domains

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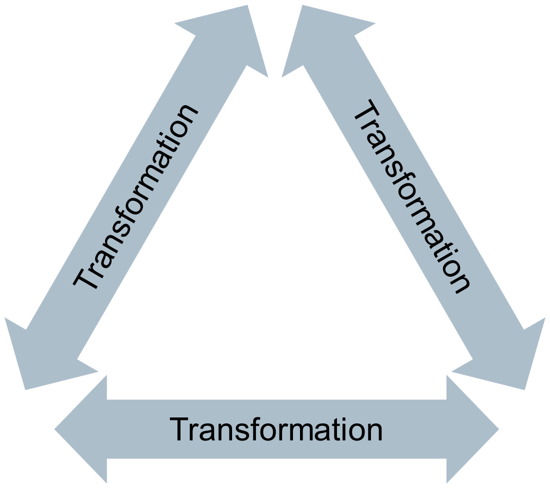
Each domain = different coordinate system that is used to view or describe the characteristics of a system or event

Time domain



Frequency domain

Amplitude domain

Each domain highlights a particular aspect of the characteristics of a system or event

- The **time domain** is usually the basis for a description of a system’s dynamic behavior. e.g. differential equation of motion. Events are measured as a function of time
- The **frequency domain** highlights the periodic characteristics of the system or event
- The **amplitude domain** represents looks at the probability distribution of the amplitudes

Fourier transformation
Joseph did help us a lot…

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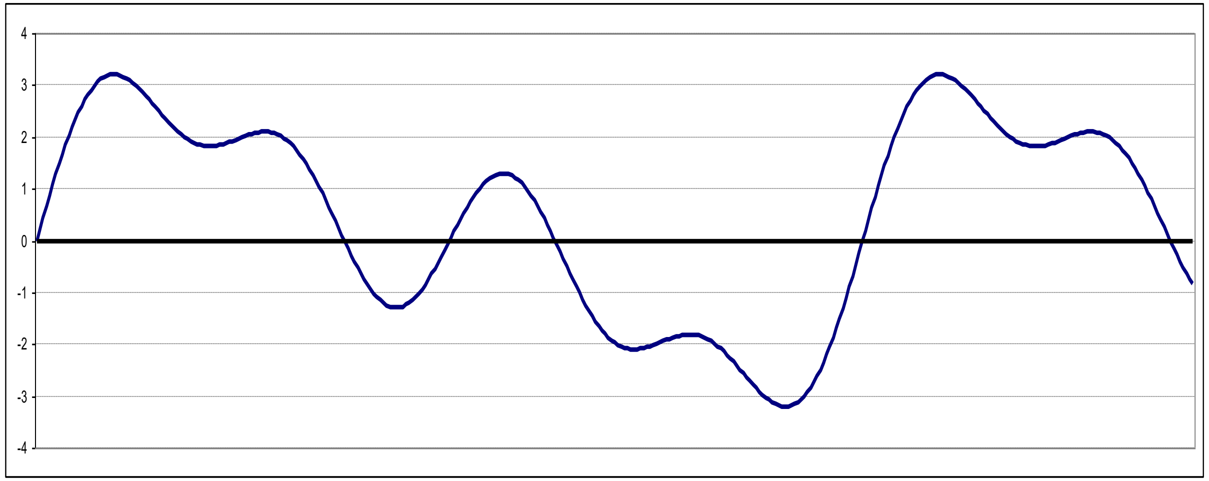
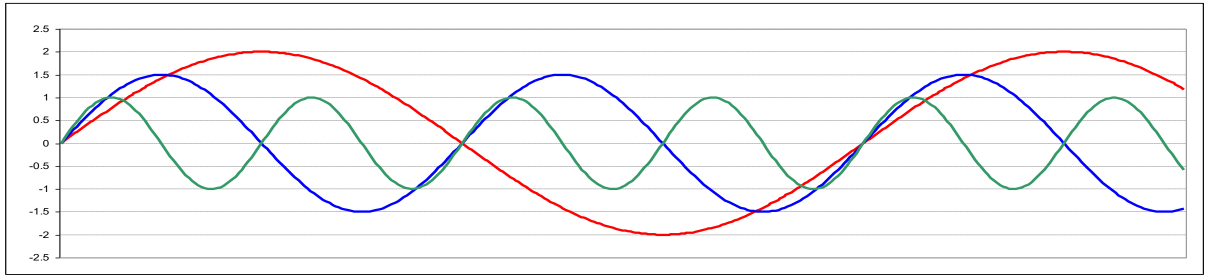
Joseph Fourier
(°1768 - †1830)
Théorie analytique de la chaleur (1822)

Fourier's law of heat conduction

$$\frac{\partial u}{\partial t} = k\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

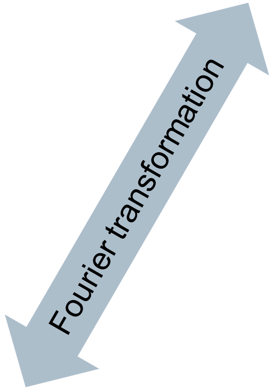
Analysed in terms of infinite mathematical series





Any signal can be described as a combination of sine waves of different frequencies

Time domain



Frequency domain

Fourier transformation

For mathematicians …

- Convert from time to frequency domain and back
- Fourier integral
- No information is lost when converting!

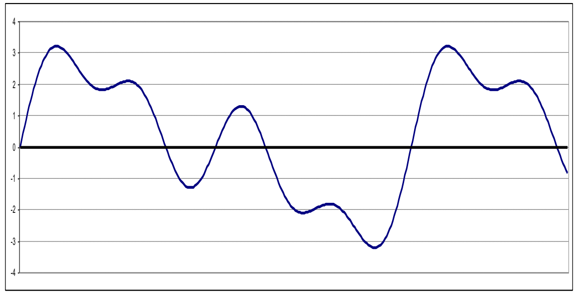
$$X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

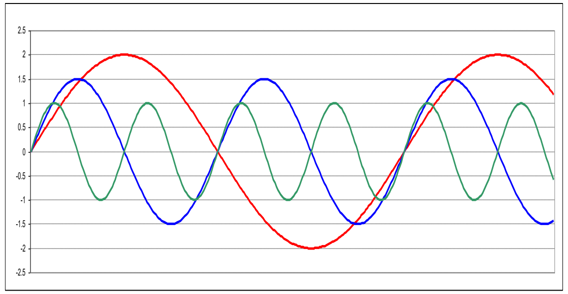
$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

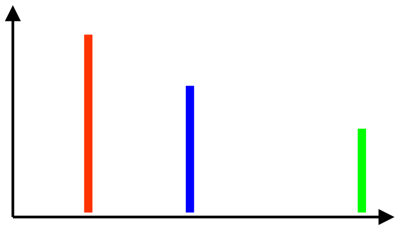
For engineers …

Time domain

Frequency domain





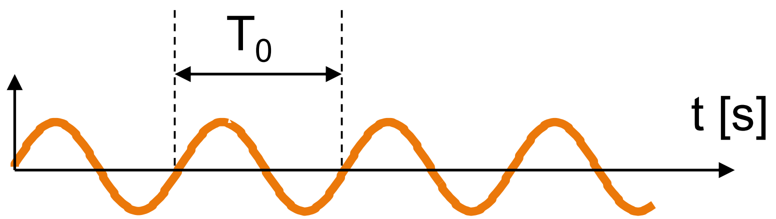


Detect sine waves

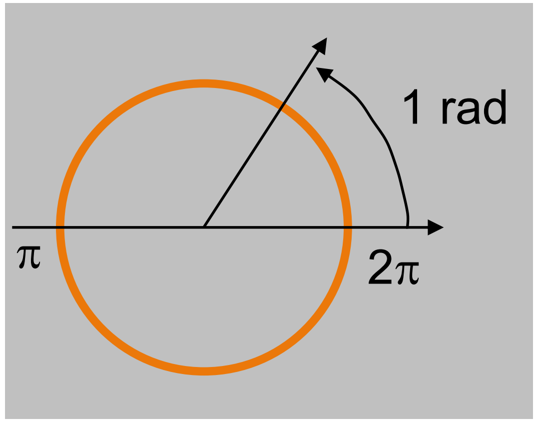
Draw line at sine frequency

Some definitions of sine waves

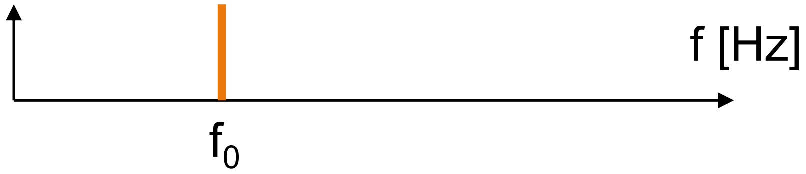
Time domain



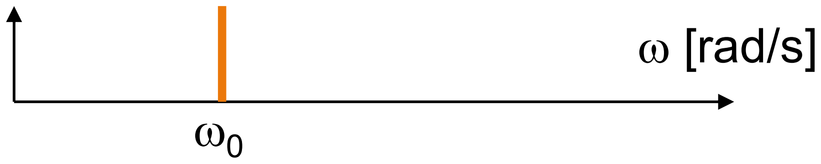
Period: T_0 [s]



Frequency domain

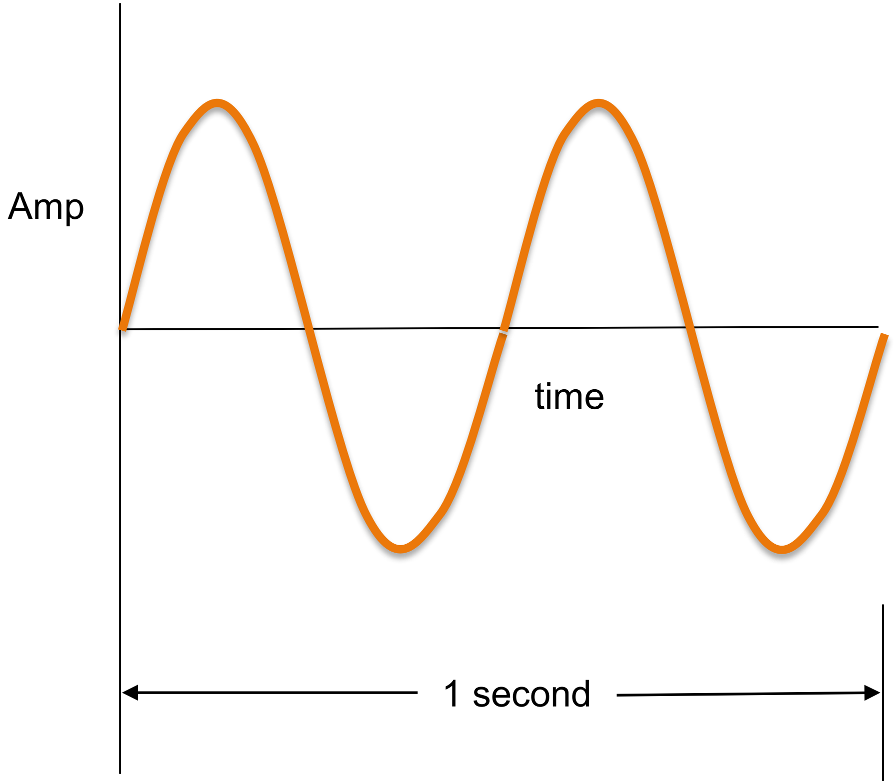


Frequency: $f_0 = 1/T_0$ [Hz]



Pulsation / circular frequency: $\omega_0 = 2\pi f_0 = 2\pi/T_0$ [rad/s]

Basics of sine waves



Sine Wave Equation

$$\boxed{x(t) = A\sin(2\pi f t + \theta)}$$

A = Amplitude

f = Frequency

θ = Phase

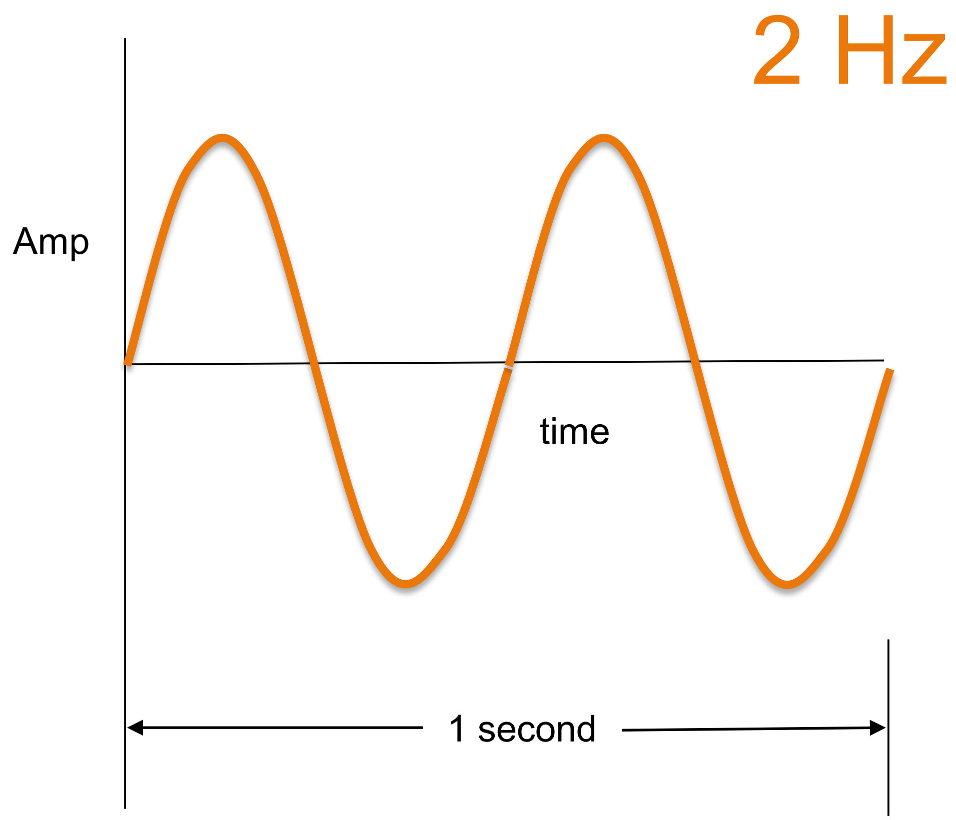
t = Time

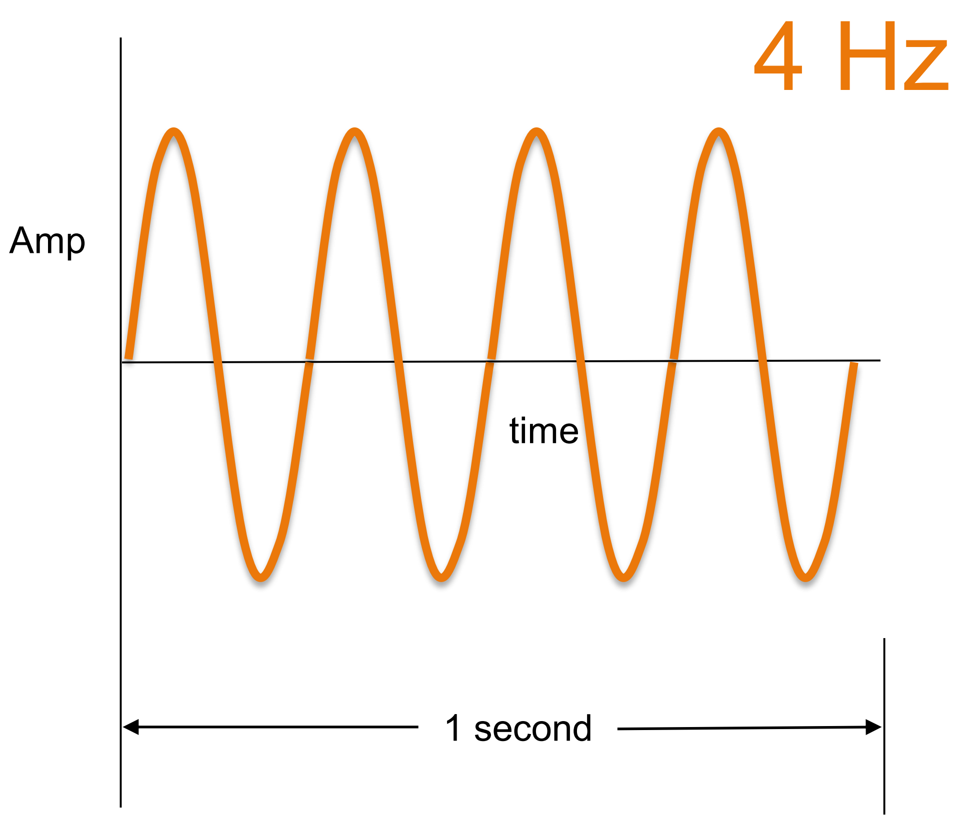
$$\boxed{\omega = 2\pi f}$$

f is in Hz

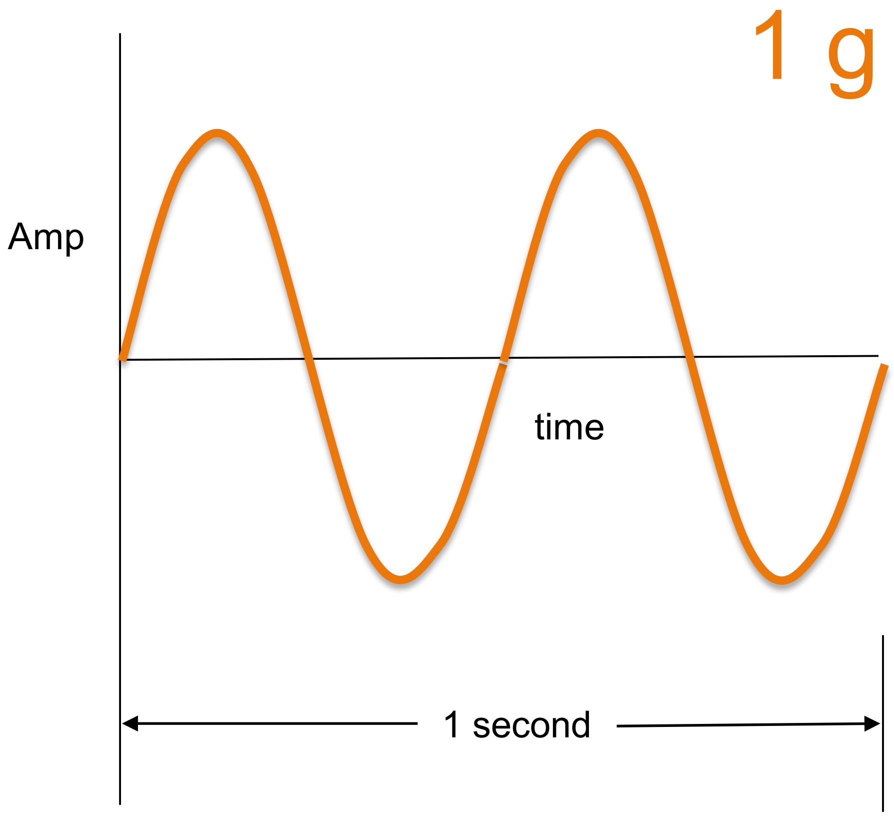
ω is in radians/sec

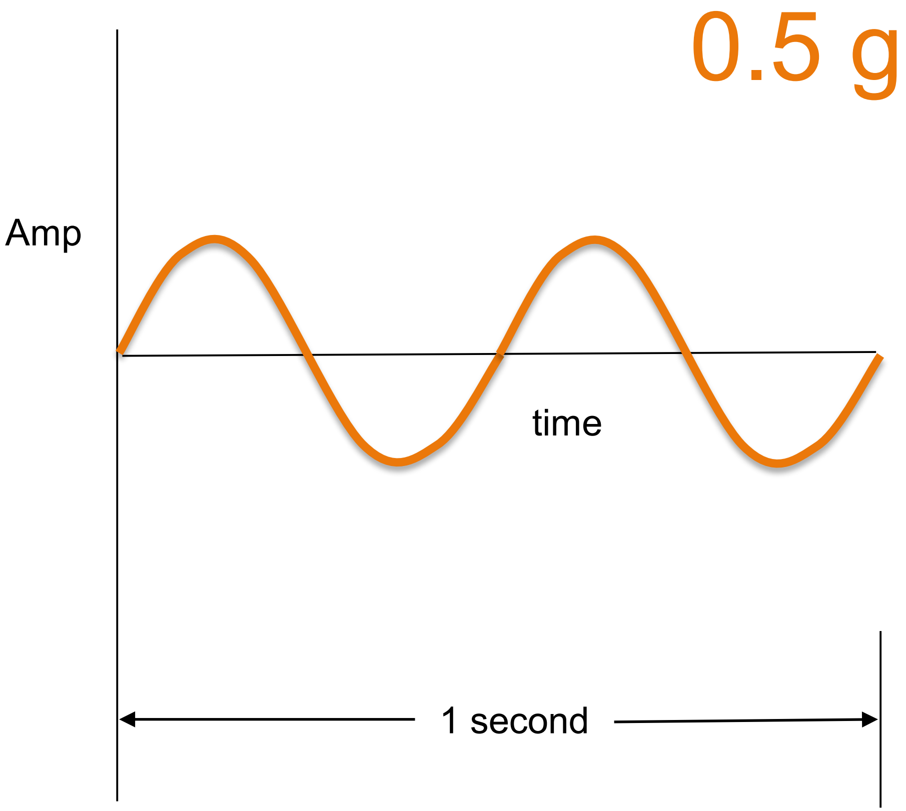
Basics of sine waves
Frequency



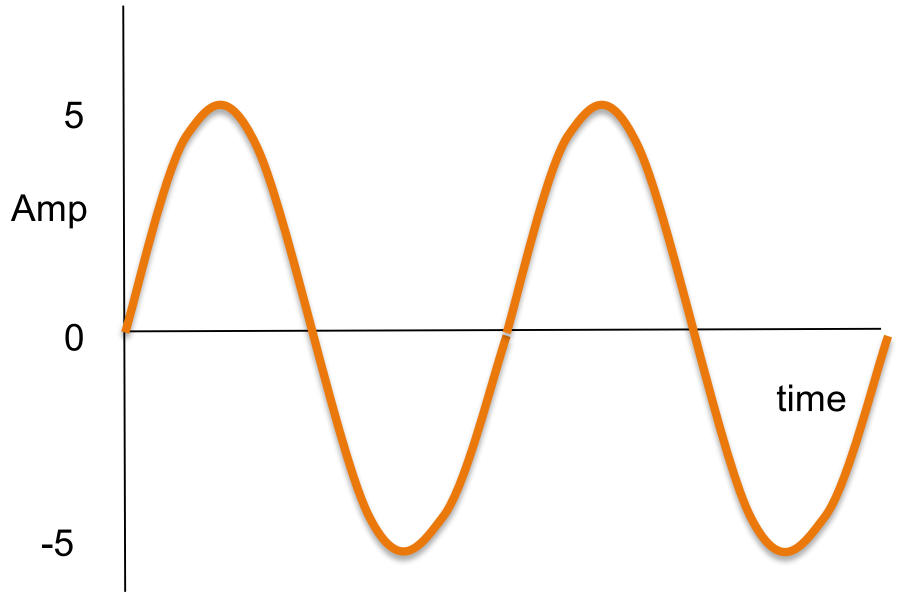


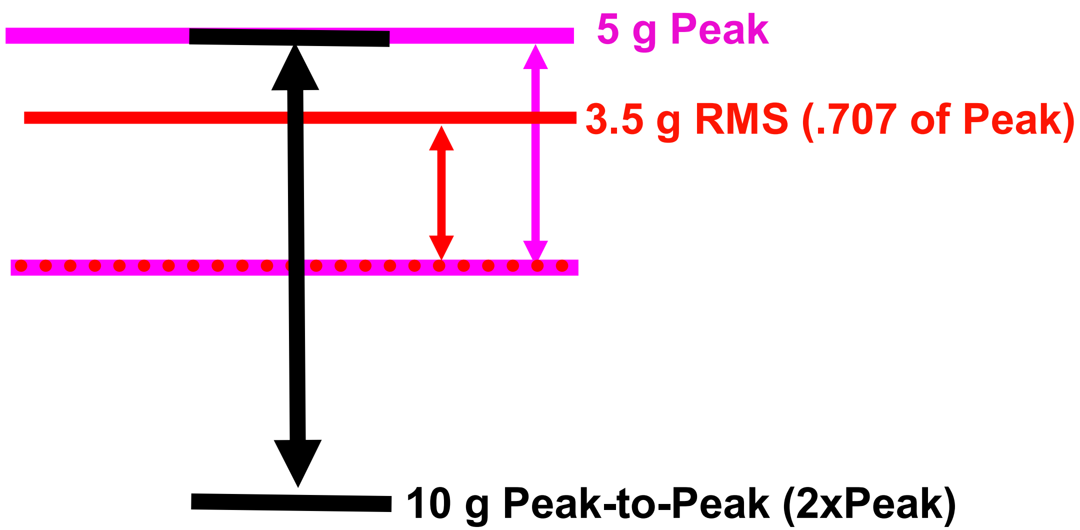
Basics of sine waves
Amplitude





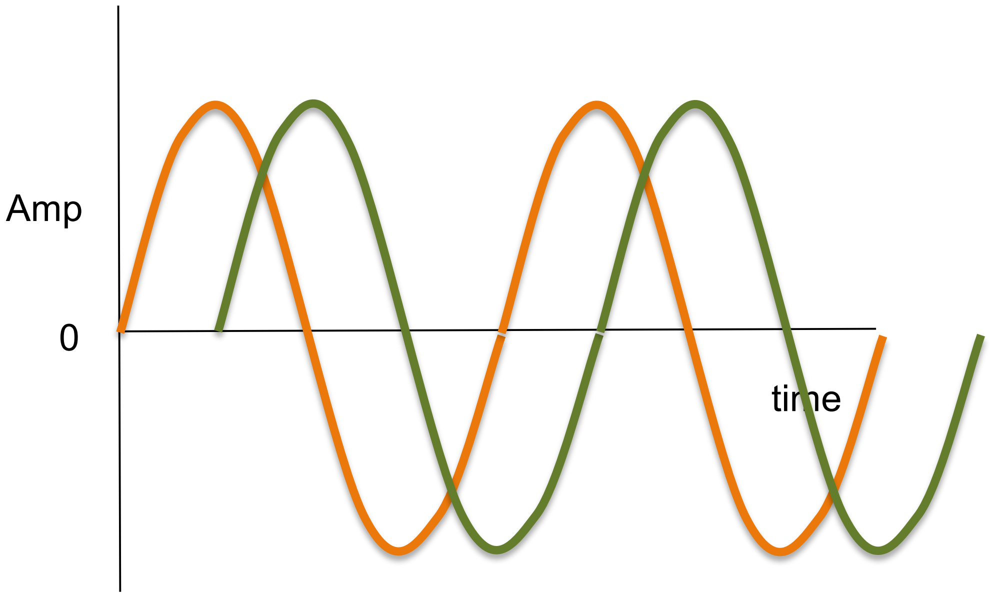
Basics of sine waves
Amplitude





Scaling can cause amplitude difference!

Basics of sine waves
Phase



$$x(t) = A\sin(2\pi f t + \underline{\theta})$$

Phase is the amount of shift in time relative to another reference
(another signal, start of FFT block, etc.)

Phase is measured as an angle
Orange signal “lags” the green by about
45° or $\pi/4$ radians
Green signal “leads” the orange by about
315° or $7\pi/4$ radians

Fourier transformation

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Q: What is the output of a Fourier transformation?

A: A spectrum in the frequency domain. It represents a series of sines and cosines in the form of complex numbers. When these numbers are summed, they form the original signal in the time domain.

Solution:

a + jb = complex number

a = real part

b = imaginary part

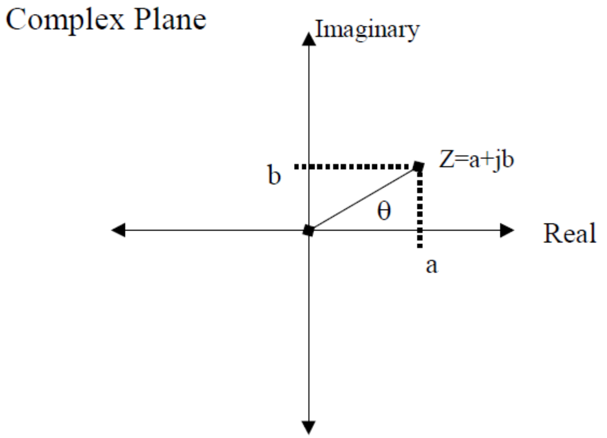
j = √-1

Note: You will get a complex number for each point in your spectrum

Frequency spectrum
Complex numbers

Q: How do I make sense of this complex numbo-jumbo?

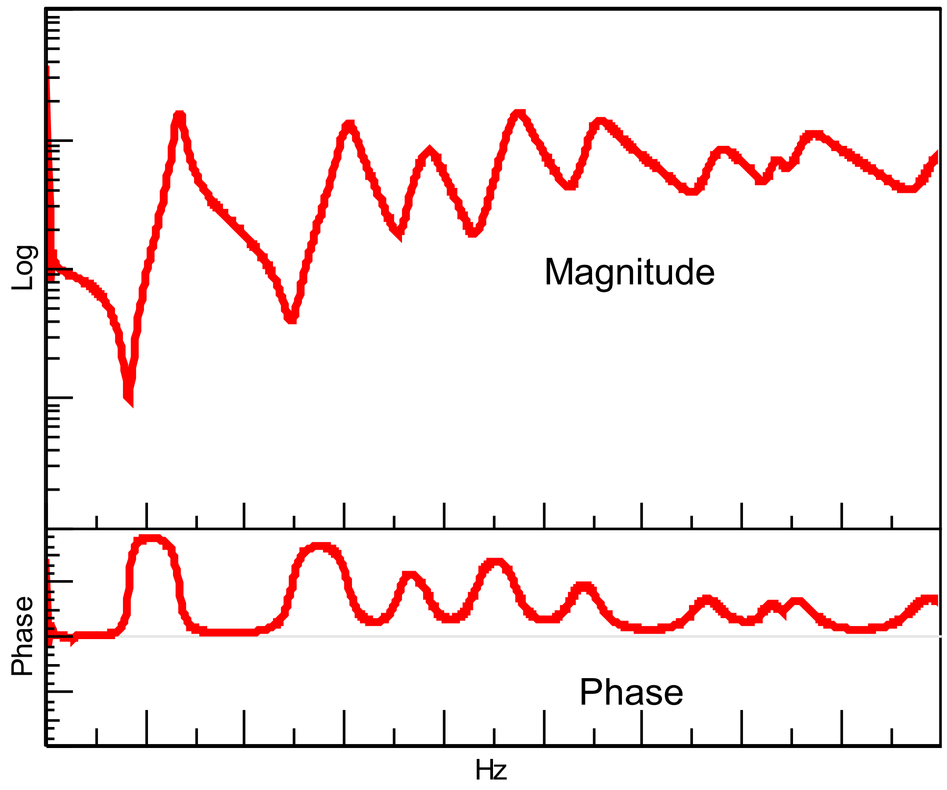
A: Spectrum most commonly viewed as **Magnitude/Phase**



$$z = (a^2 + b^2)^{1/2} = [(a + jb)*(a - jb)]^{1/2} = \text{magnitude}$$

$a = z * \cos\theta$ (magnitude * cosine of the phase)
$b = z * \sin\theta$ (magnitude * sine of the phase)

$$\tan\theta = \frac{z * \sin\theta}{z * \cos\theta} = \frac{b}{a} \qquad \theta = \tan^{-1}(b/a)$$



AKA: Bode Plot

Frequency spectrum
Time history

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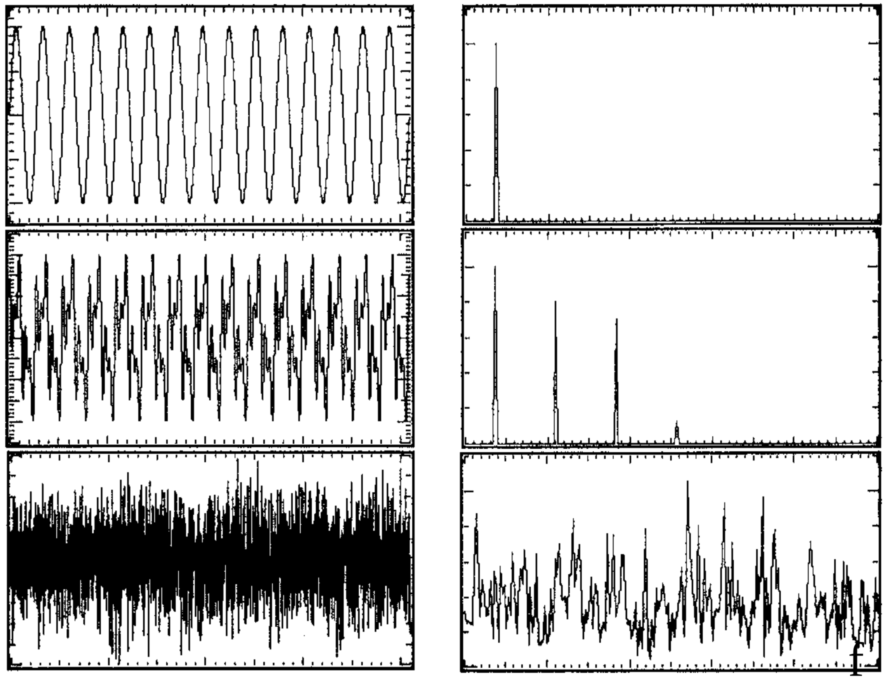
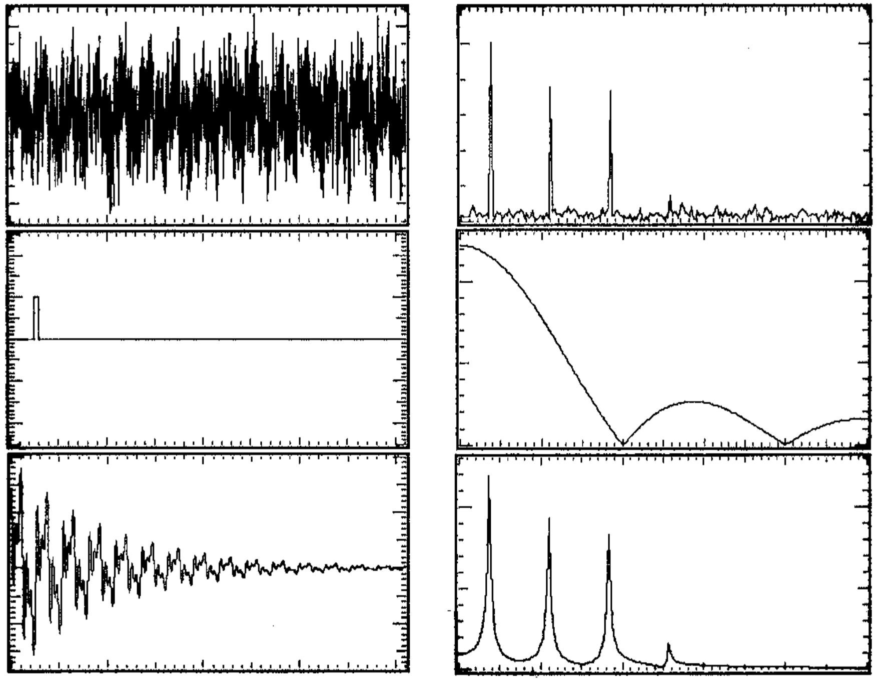
Selection of domain depends on the application aims
Equivalence of time and frequency domain: no loss of information



Time **Frequency**



Time **Frequency**

Signals

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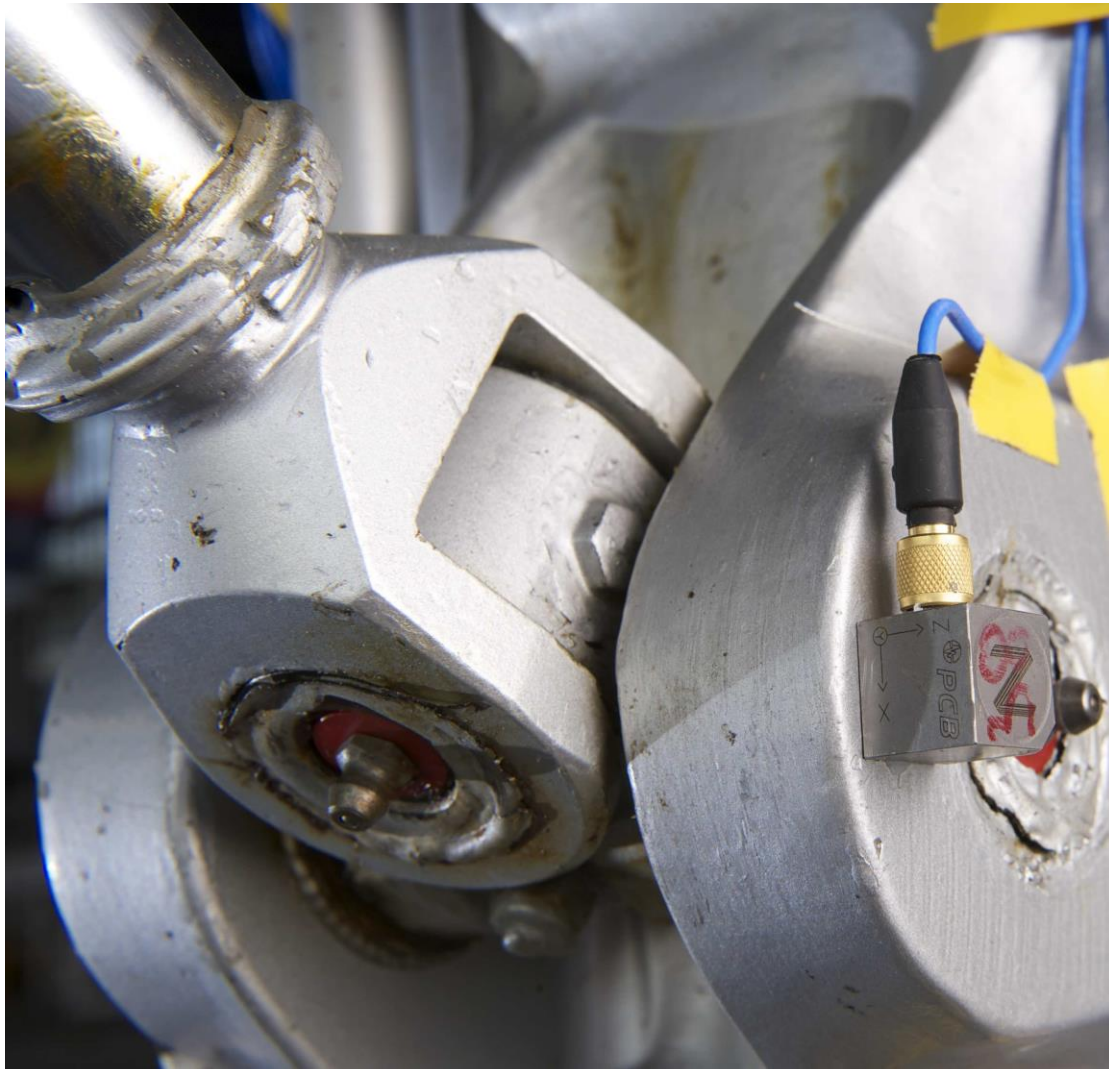

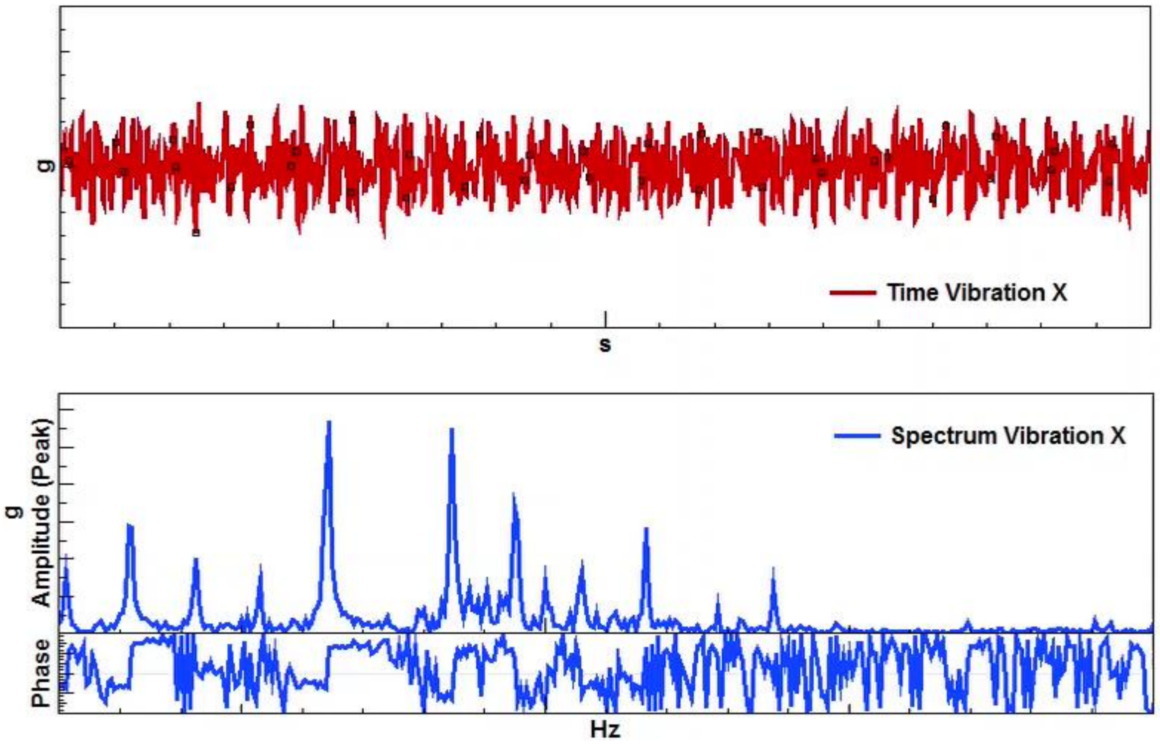

To be continued….

PART 2: Digitizing signals

- Sampling
- Aliasing

PART 3: Effects to be aware of when converting to digital

- Quantization
- Leakage

Counter-measures to ensure your digital data is valid

Thank you

Fundamentals of DSP Part 2

Introduction to Digital Signal Processing

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Part 1: What is a signal?

Time and frequency domain

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- **Aliasing**

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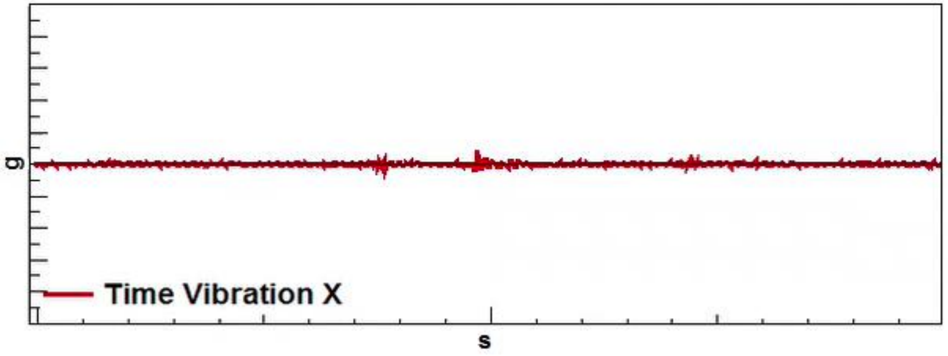
Counter-measures to ensure your digital data is valid

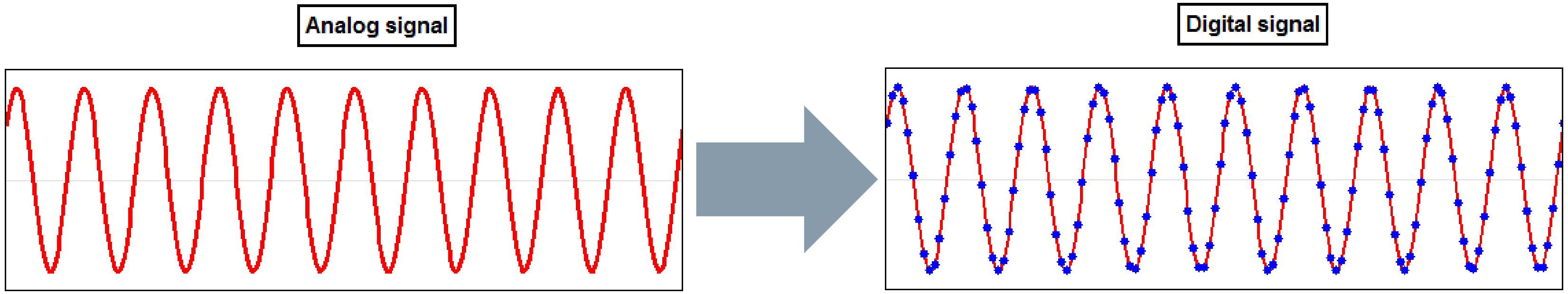
Nice theory…

but we need to do this on a computer

Digital Signal Processing: apply manipulations using a computer-based system

- Most transducers output an analog (continuous) signal
- Computers are digital devices (0/1; on/off)
- Convert the sensor signal into a discrete stream of digital information
- Discretization in time and in amplitude
- Massive loss of information when sampling





Sampling analog signals

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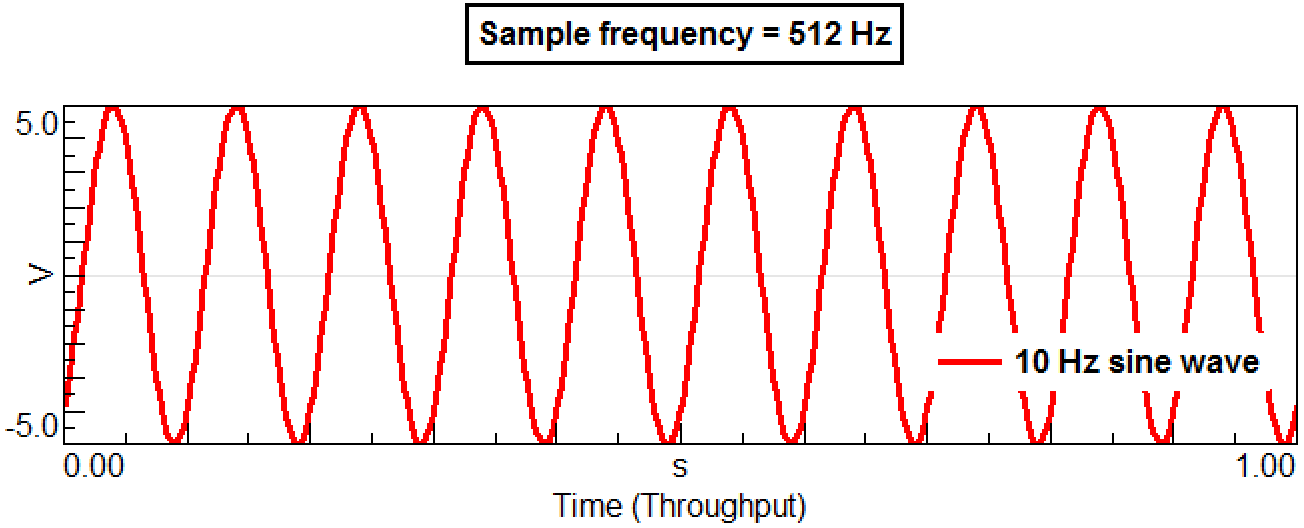
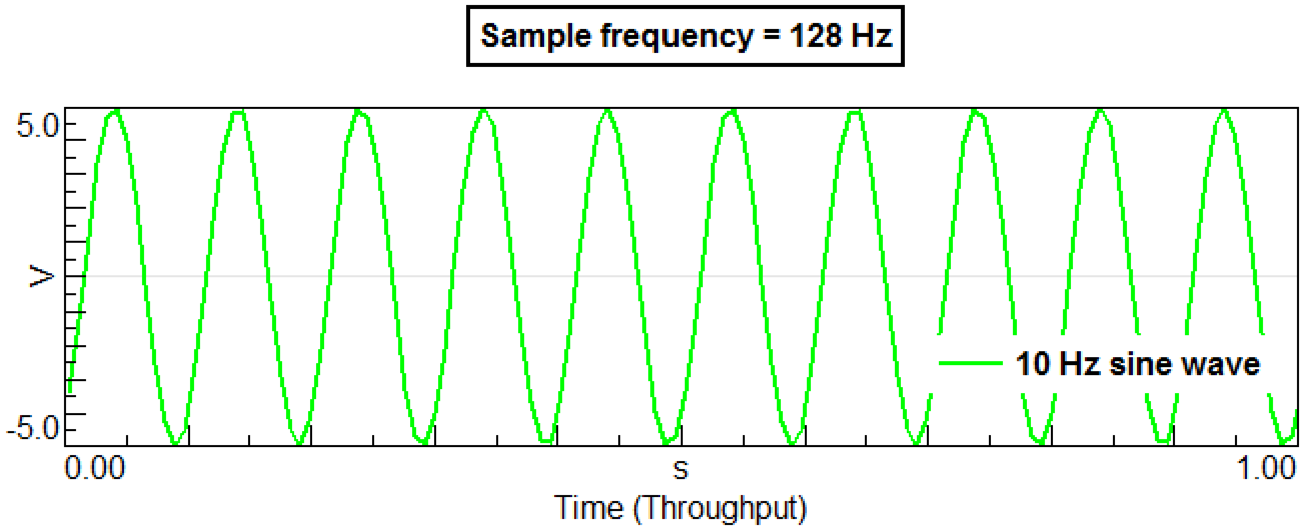


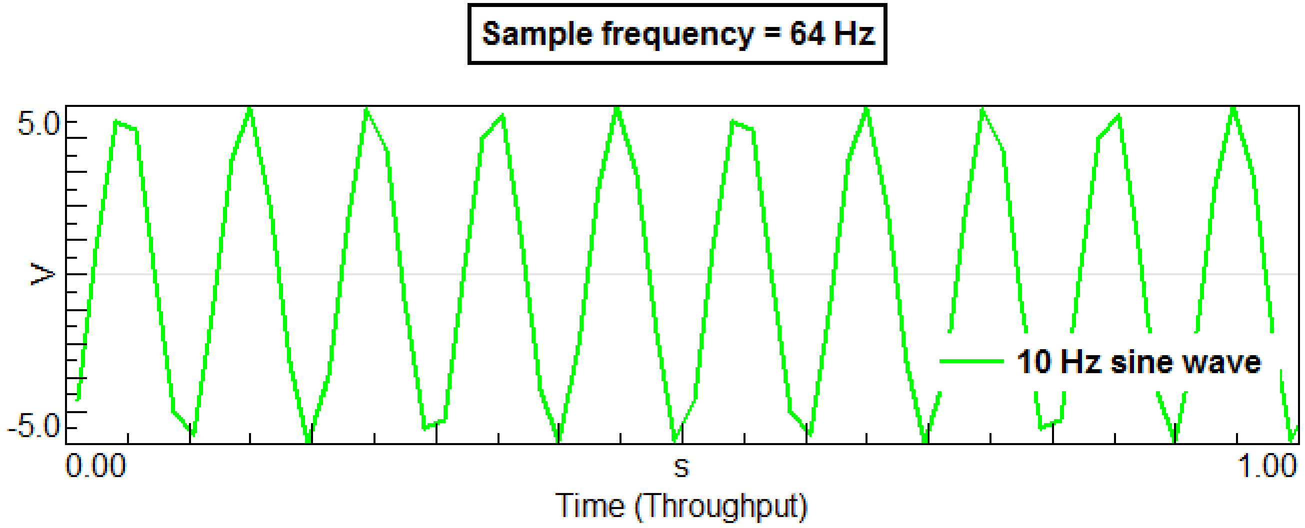
10 Hz sine wave, sampled at 512 Hz: digital representation looks like a perfect sine

Sampling analog signals



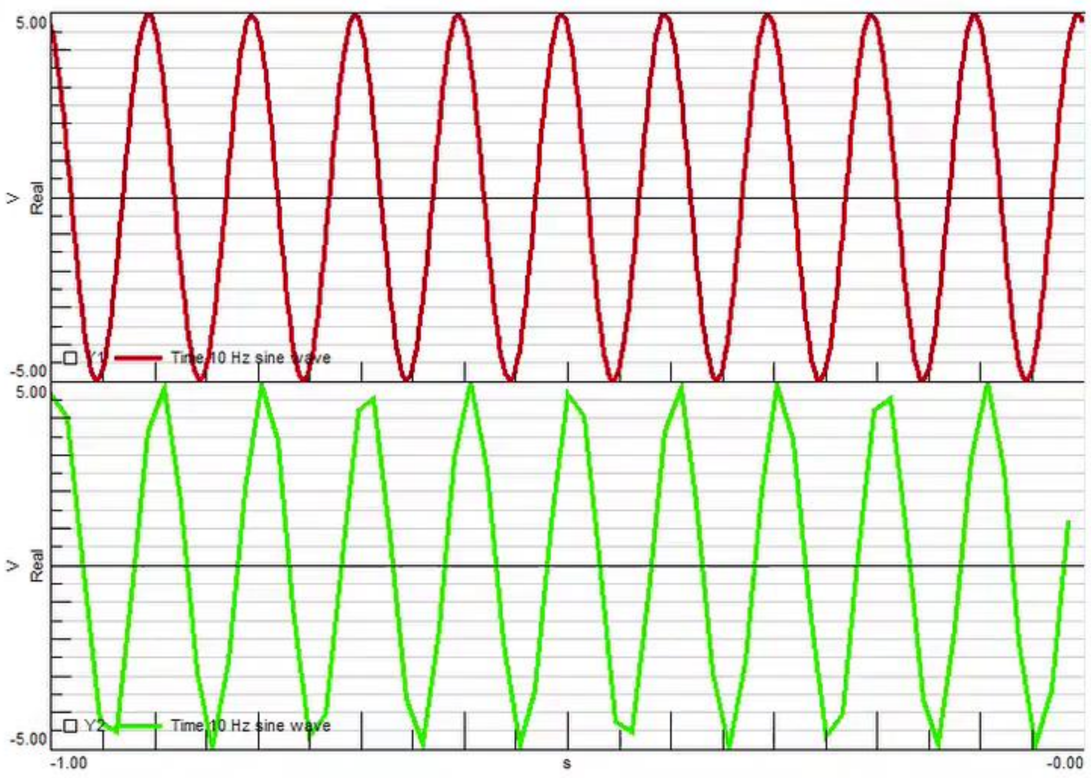
10 Hz sine wave, sampled at 128 Hz: digital representation still looks OK

Sampling analog signals



10 Hz sine wave, sampled at 64 Hz: digital representation starts looking strange…

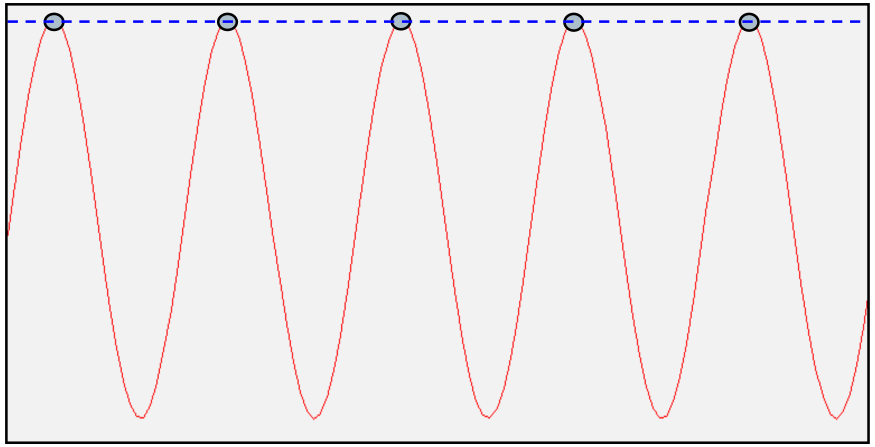
Sampling analog signals



10 Hz sine wave sampled high (512 Hz, red) and low (64 Hz, green) on a digital oscilloscope

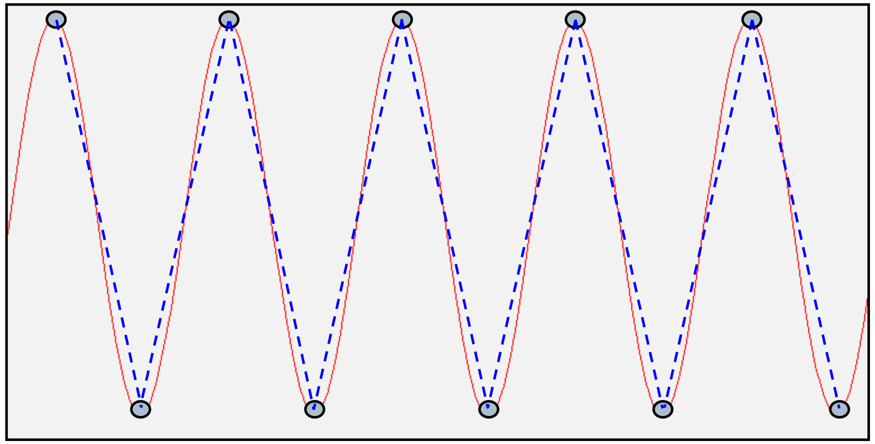
Sampling analog signals
Exploring the limits…

Sampling frequency = sine wave frequency
$f_s = f_{sine}$



Observed frequency = 0 Hz (DC)

Sampling frequency = 2 x sine wave frequency
$f_s = 2 \times f_{sine}$

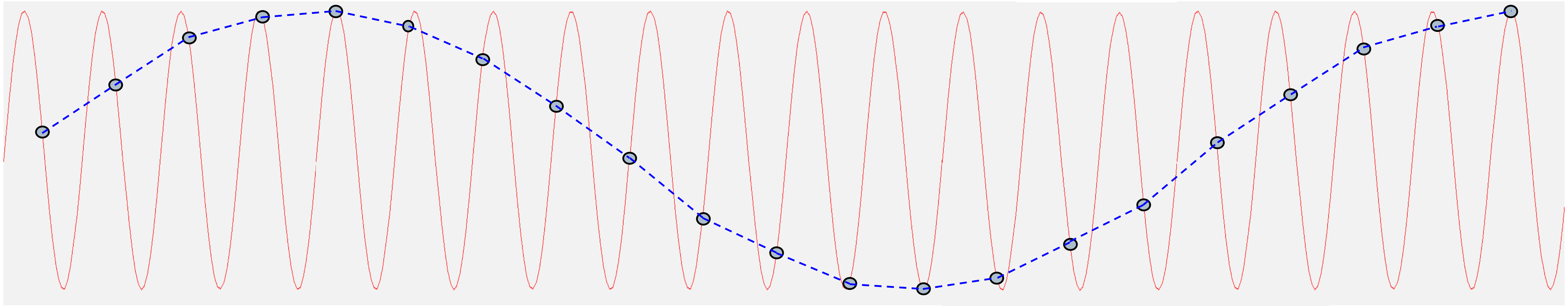


Observed frequency is correct, but it is borderline
(sampling frequency cannot be lowered)

Sampling analog signals

When pushing further, you get aliasing

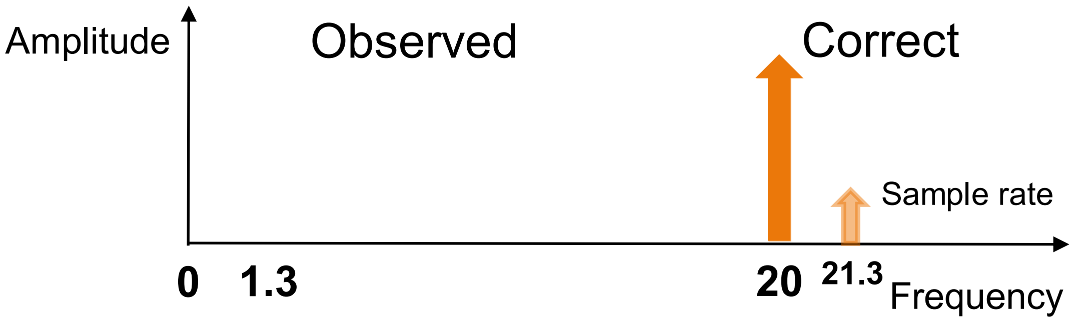
Sine wave frequency = 20 Hz
Sample rate 21.3 Hz



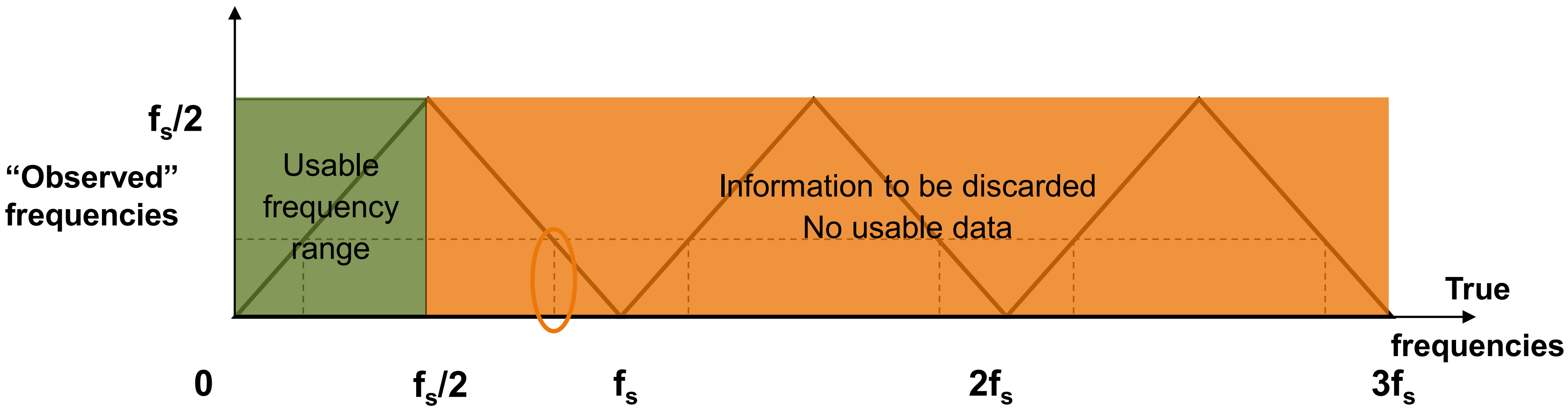
Observed frequency is wrong: 20 Hz sine wave sampled at 21.3 shows as 1.3 Hz signal

Sampling analog signals

When pushing further, you get aliasing



Aliasing



Nyquist frequency

$$f_{max} = \frac{f_s}{2}$$

Sampling analog signals
How to prevent aliasing?

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Select sample rate to cover full signal bandwidth

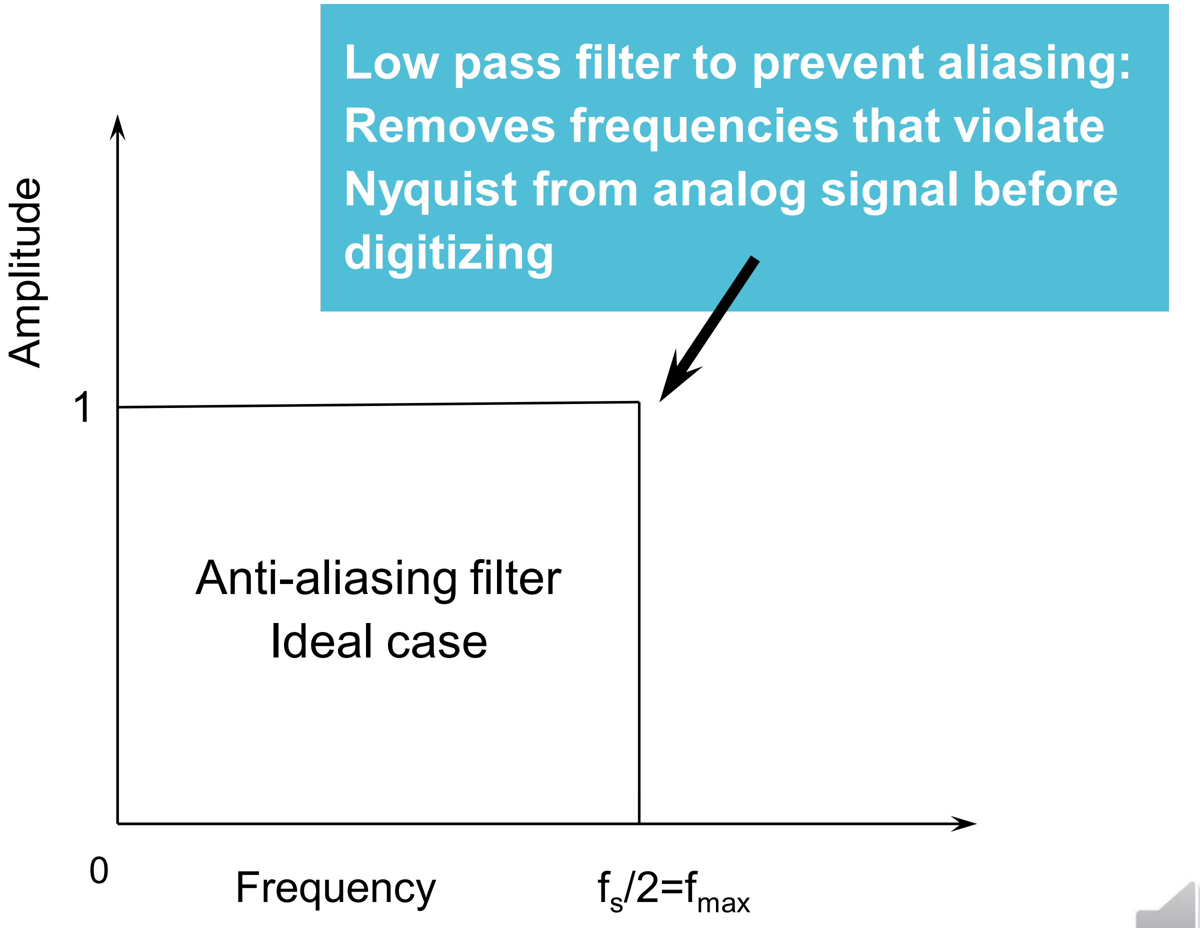
If there is no frequency content above Nyquist Frequency, then there is no Aliasing

This is not always practical or possible:

- Large files sizes
- Limitations of data acquisition equipment

Limit signal bandwidth using Anti-Aliasing Filter

- Analog and/or Digital low-pass filters



Sampling analog signals
How to prevent aliasing?

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Make sure the signal does not contain frequencies above half the sample frequency

Do this by applying a sufficient performing low-pass filter

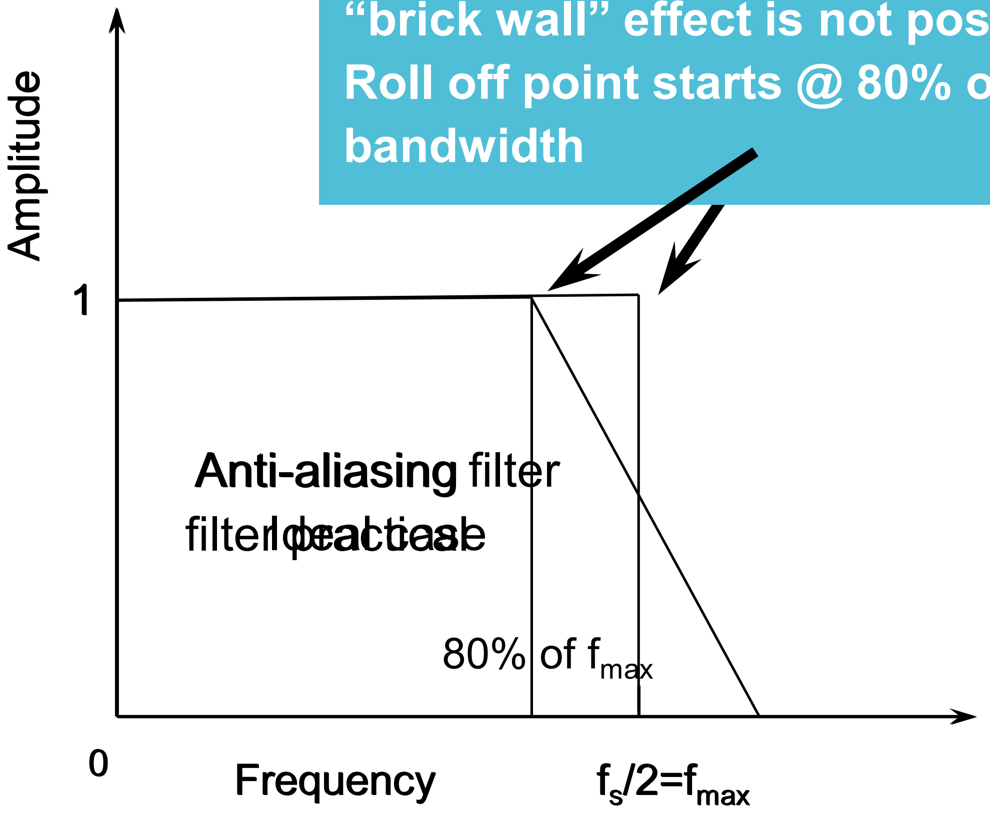
Limit signal bandwidth using Anti-Aliasing Filter

Be aware that the amplitude of the last portion of the spectrum is

- Analog and/or Digital low-pass filters

Automatically done in good data acquisition hardware

Extremely sharp analog filter, but "brick wall" effect is not possible
Roll off point starts @ 80% of bandwidth



Aliasing demonstration

Sine sweep from 45 to 82 Hz, sampled at 128 Hz

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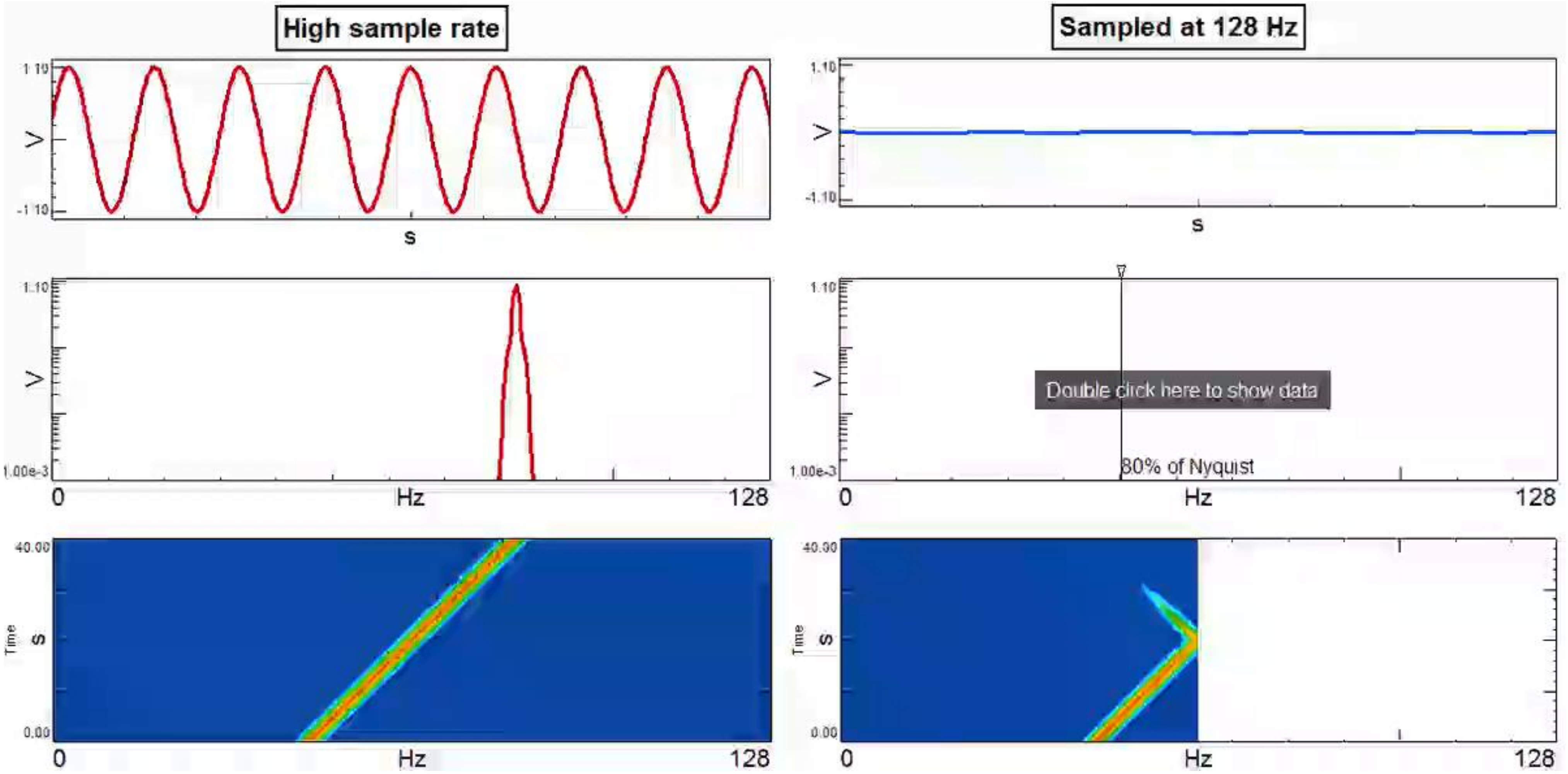

To be continued….

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PART 3: Effects to be aware of when converting to digital

- Quantization
- Leakage

Counter-measures to ensure your digital data is valid

Thank you

Fundamentals of DSP Part 3

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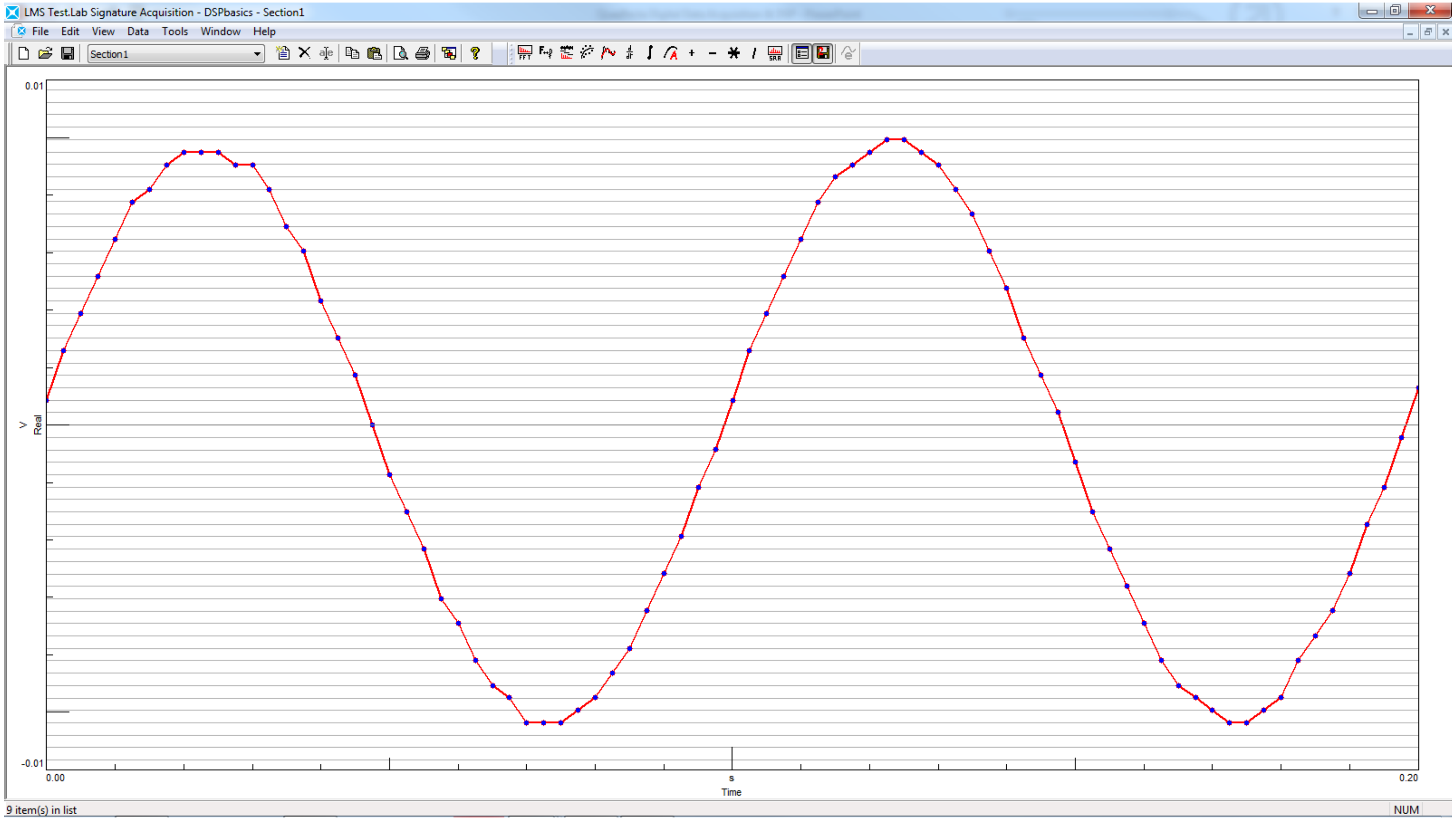
- **Leakage**

Counter-measures to ensure your digital data is valid

Digitizing analog signals
Quantization resolution



Sampling in time domain: Store an amplitude value on a periodic basis (fs)

Amplitude is determined with a discrete resolution

Real values are rounded or truncated to discrete levels

This process causes quantization noise

Digitizing analog signals
Some formulas

- Assume ADC with M bits at the output
- The number of voltage intervals N is given by

$$N = 2^M - 1$$

- The voltage resolution of an ADC is equal to its overall voltage measurement range divided by the number of discrete values:

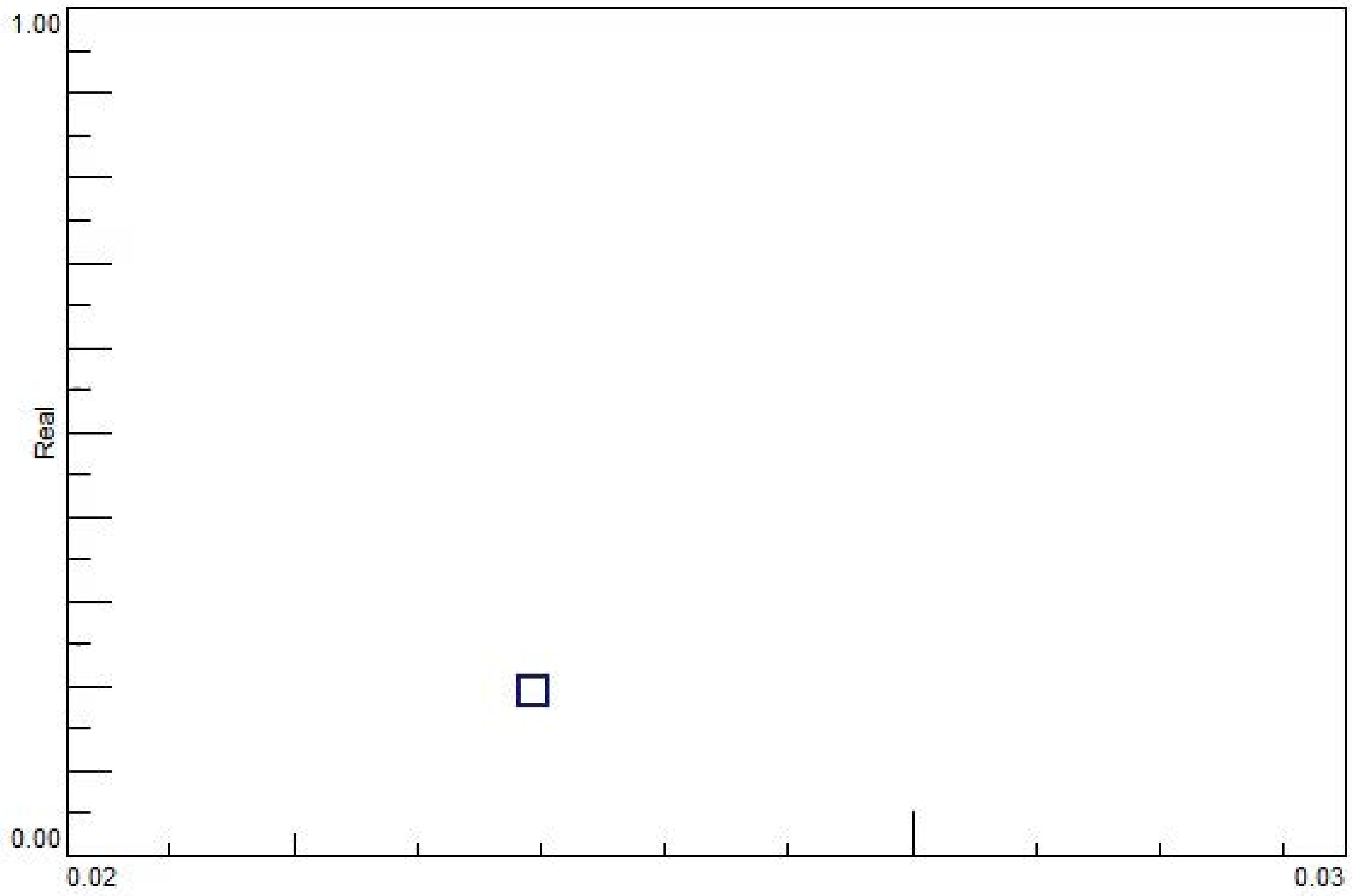
$$\Delta V = \frac{V_{range}}{2^M}$$

- The Signal-to-quantization-noise ratio is given by

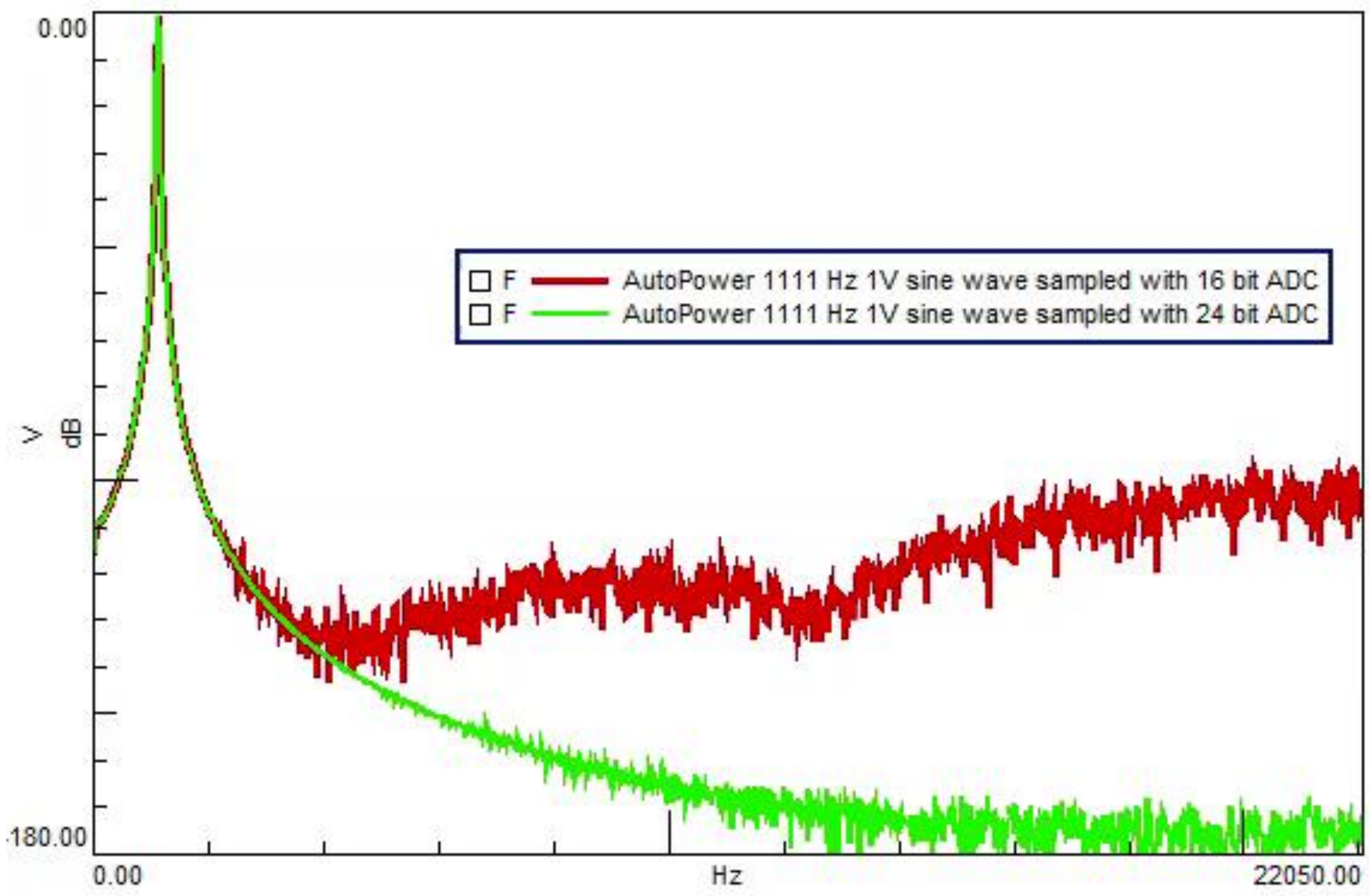
$$SQNR = 20\, log_{10}(2^M) \approx 6.02 * M\ dB$$

M	N	ΔV	SQNR
# of bits	# of voltage steps	Voltage resolution (for +/- 10V range)	Signal to quantification noise ratio
4	16	1.24 V	24 dB
8	255	78.1 mV	48 dB
12	4 095	4.88 mV	72 dB
16	65 535	0.305 mV	96 dB
24	16 777 215	1.19 µV	144 dB

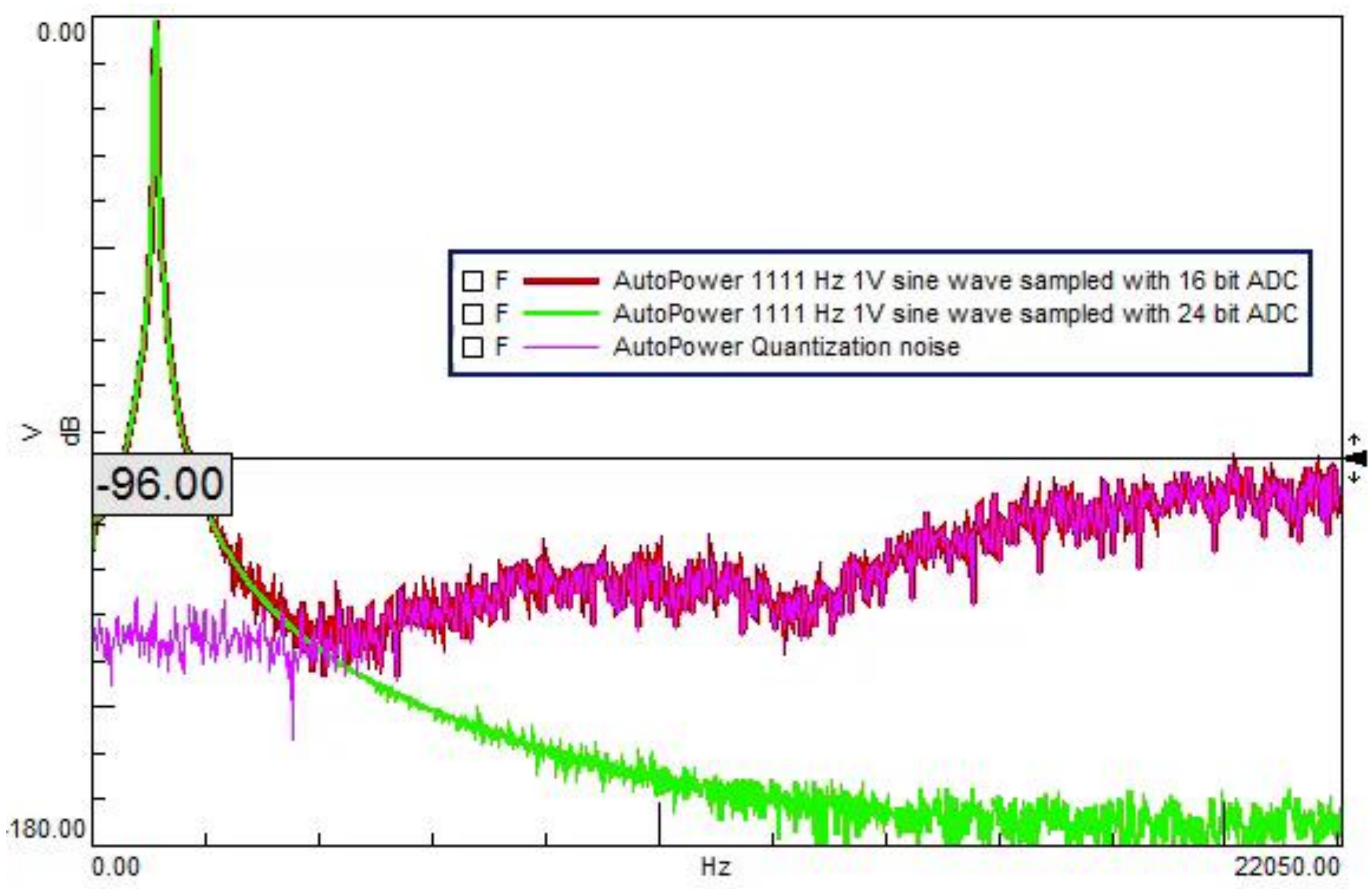
Quantization demonstration



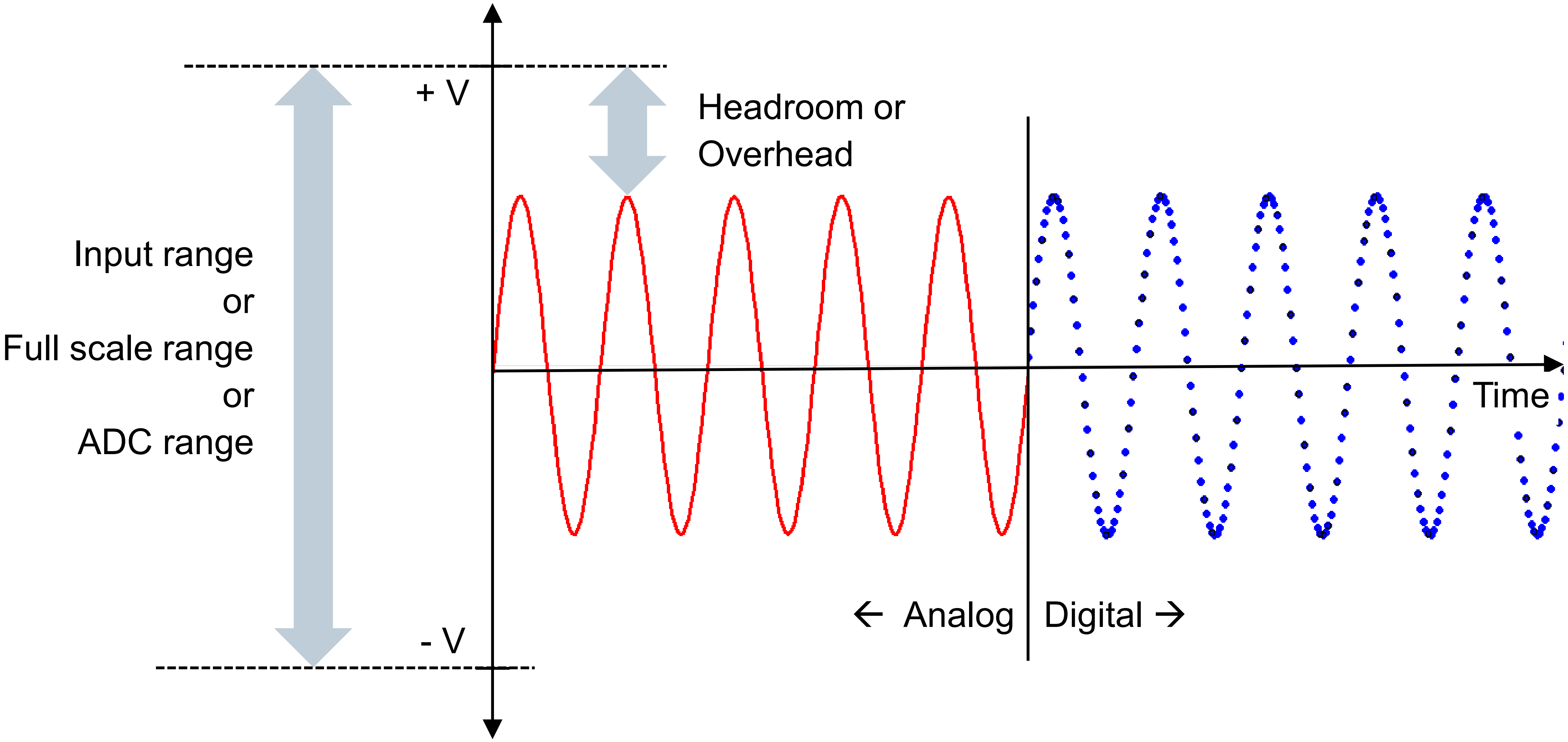
Quantization demonstration



Quantization demonstration



Mitigating quantization errors
Some terminology

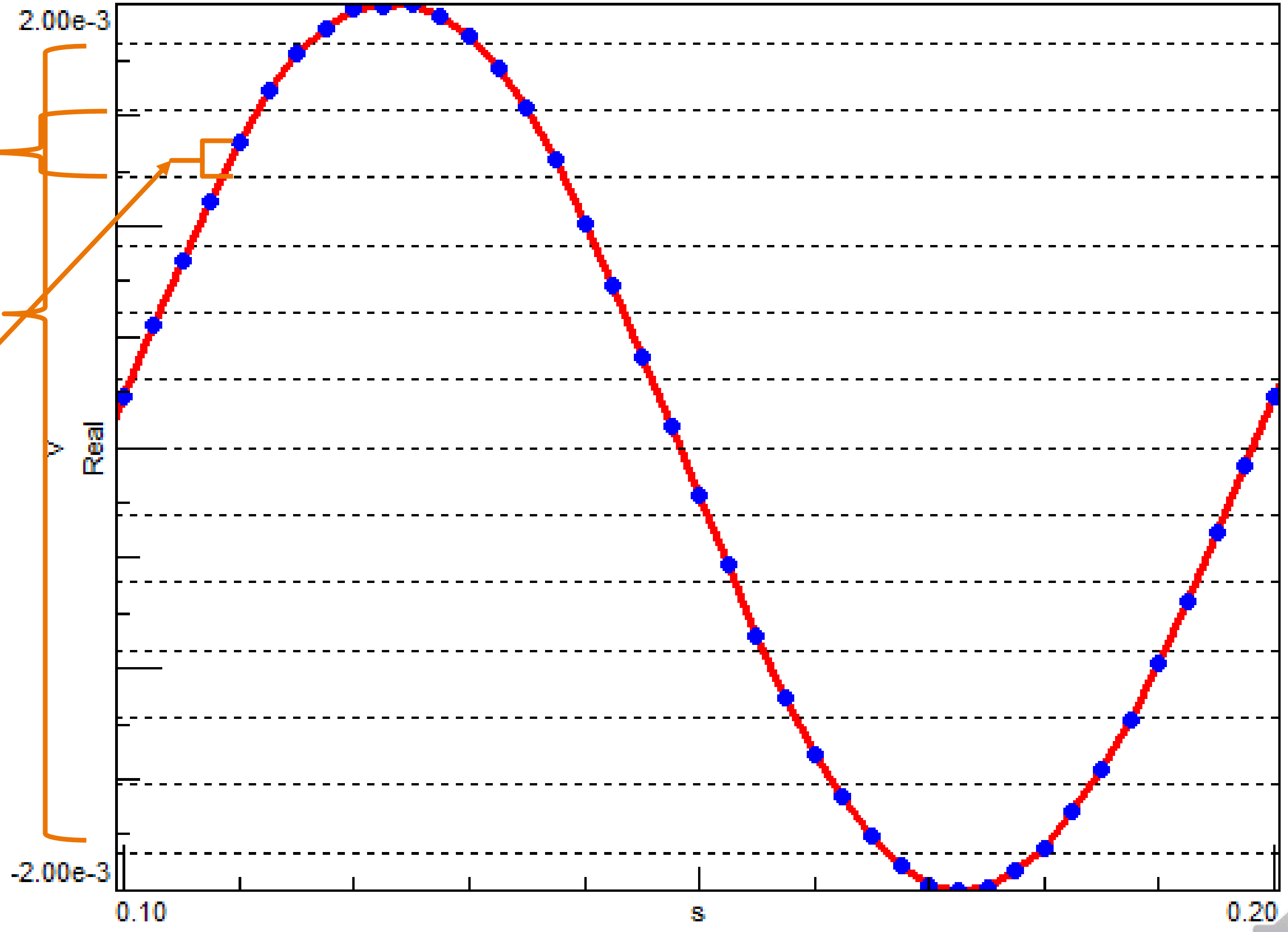


Mitigating quantization errors
Demonstration: 10 Hz, 2 mV sine wave

16 bit ADC, 10 V input range
➔ 0.305 mV discretization level

2 mV signal / 0.305 mV = 13
➔ truncated to 13 possible levels

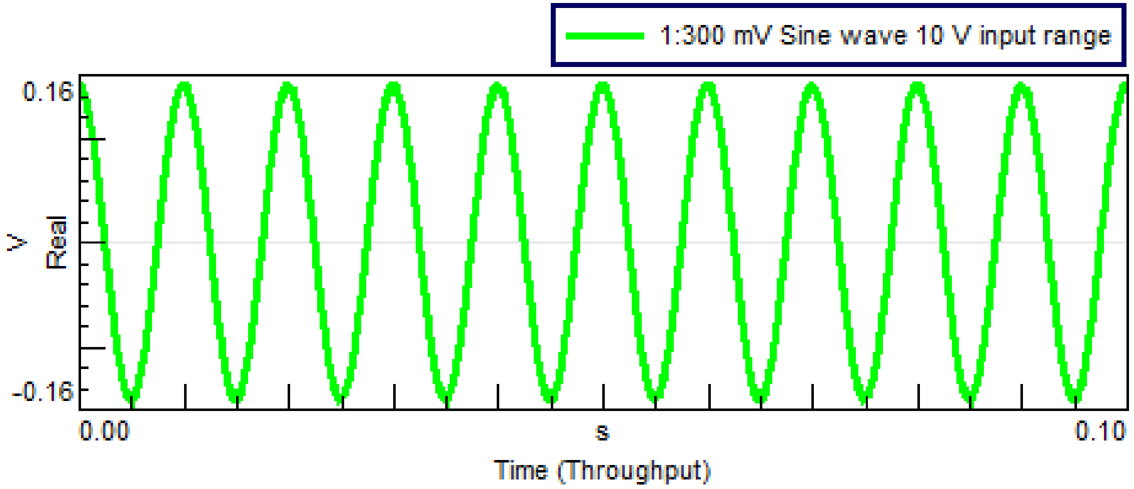
➔ Large quantization errors!

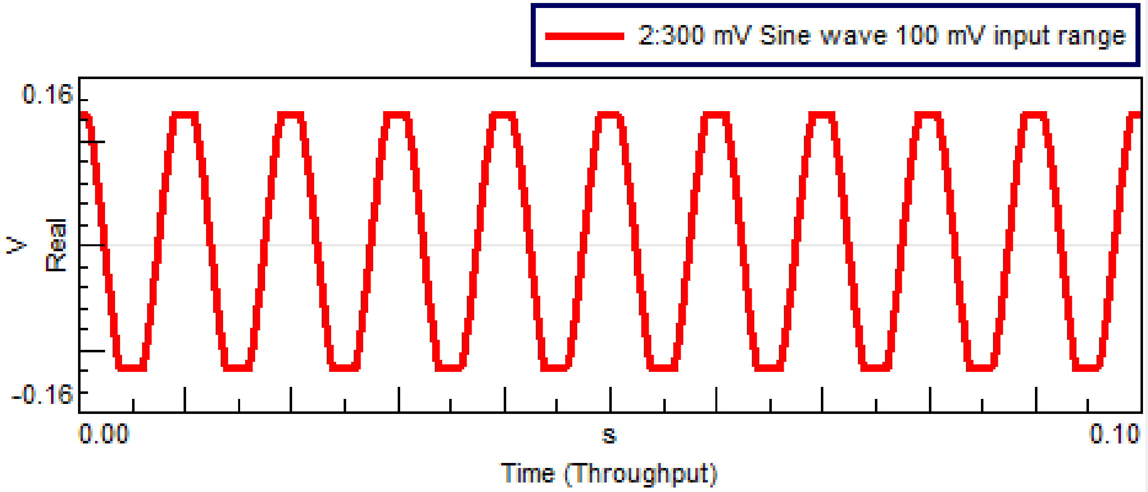


2 possible solutions:

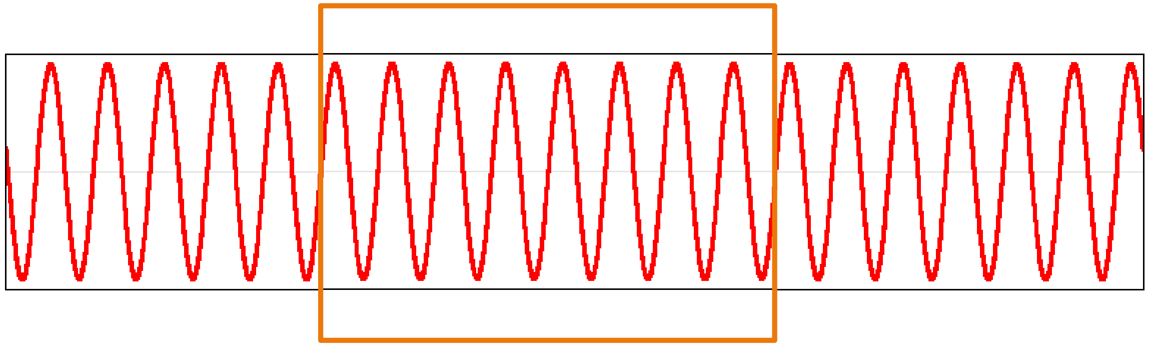
- **Use ADC with higher number of bits**
- **Apply gain to the signal prior to ADC**

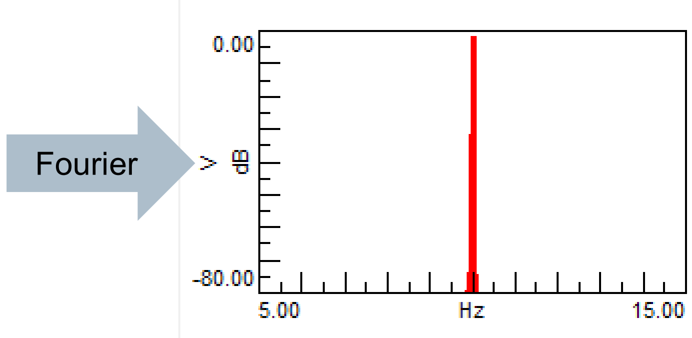
Select the right range, but watch out for overloads!





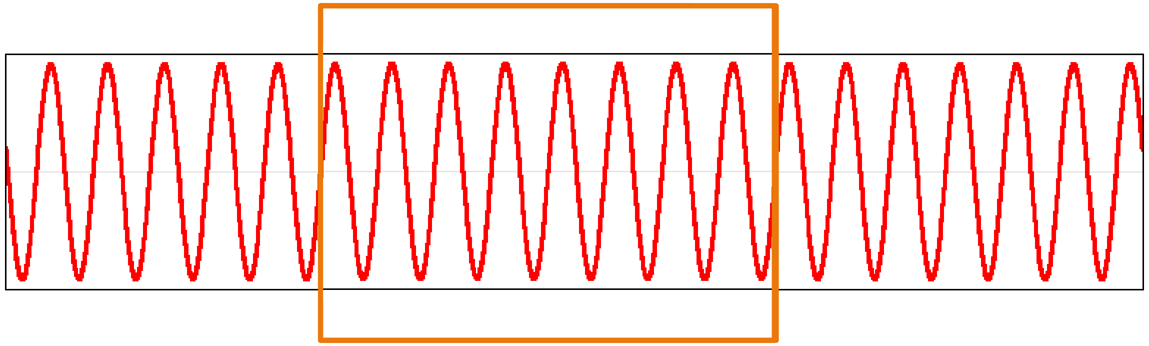
We don't have all day…
Finite observation period

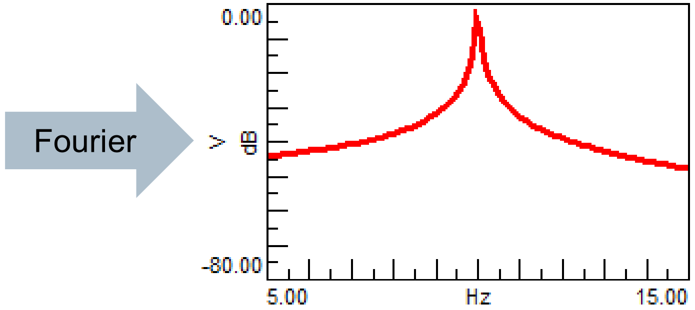




Periodic observation: correct amplitude level at correct spectral line

If signal is not periodic in observation window
➔ Leakage





Periodic observation: correct amplitude level at correct spectral line

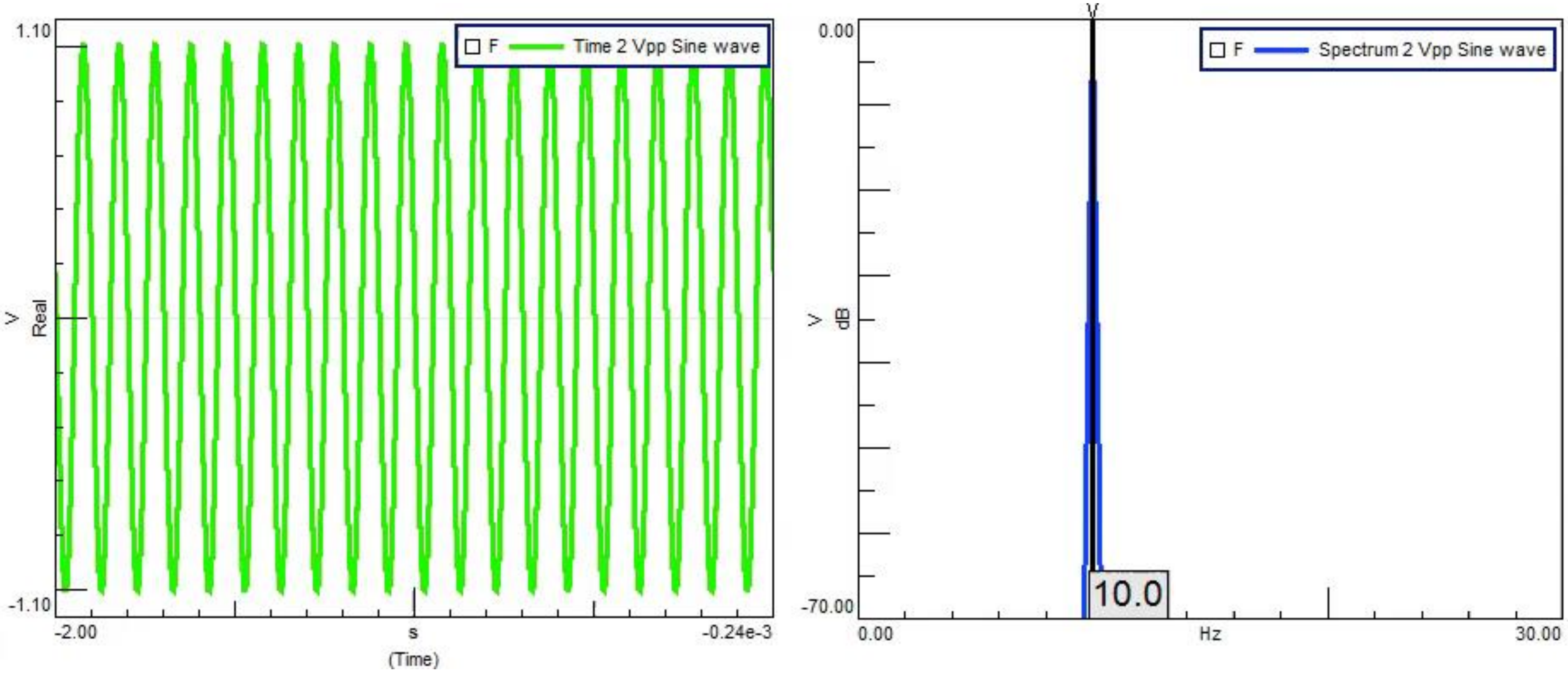
A-periodic observation: up to 63% amplitude error at spectral line closest to correct frequency

Remaining 37% amplitude spread out over entire frequency spectrum

Leakage = severe distortion of spectrum if the signal is not periodic in the observation window

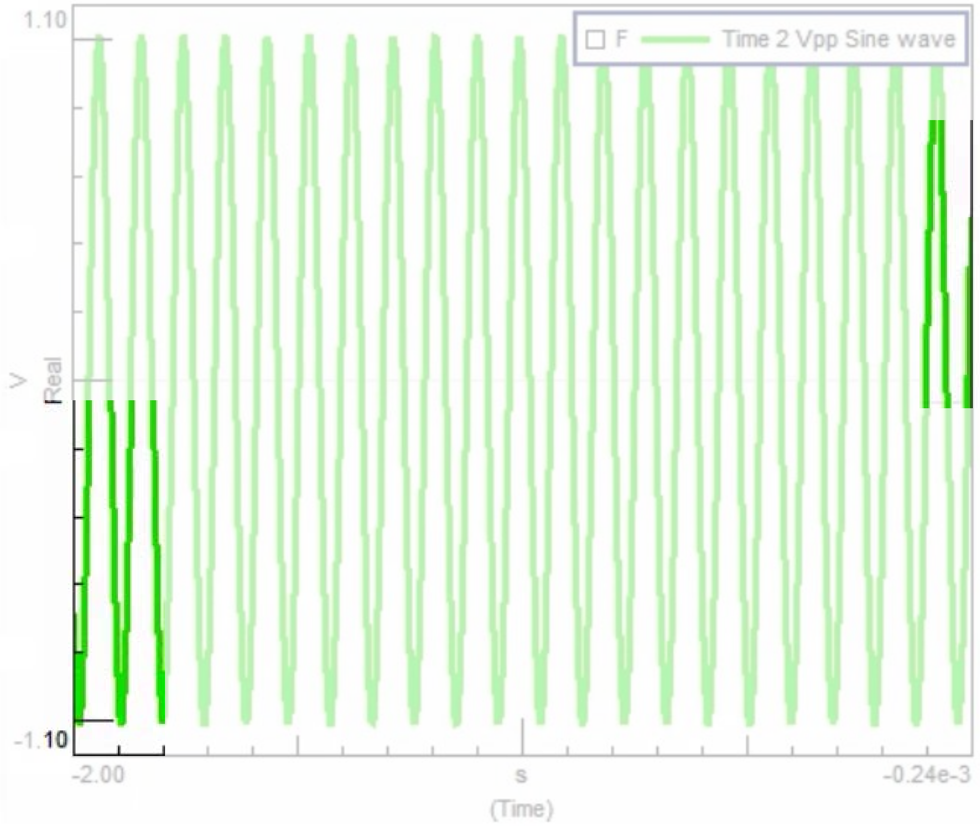
Leakage demonstration

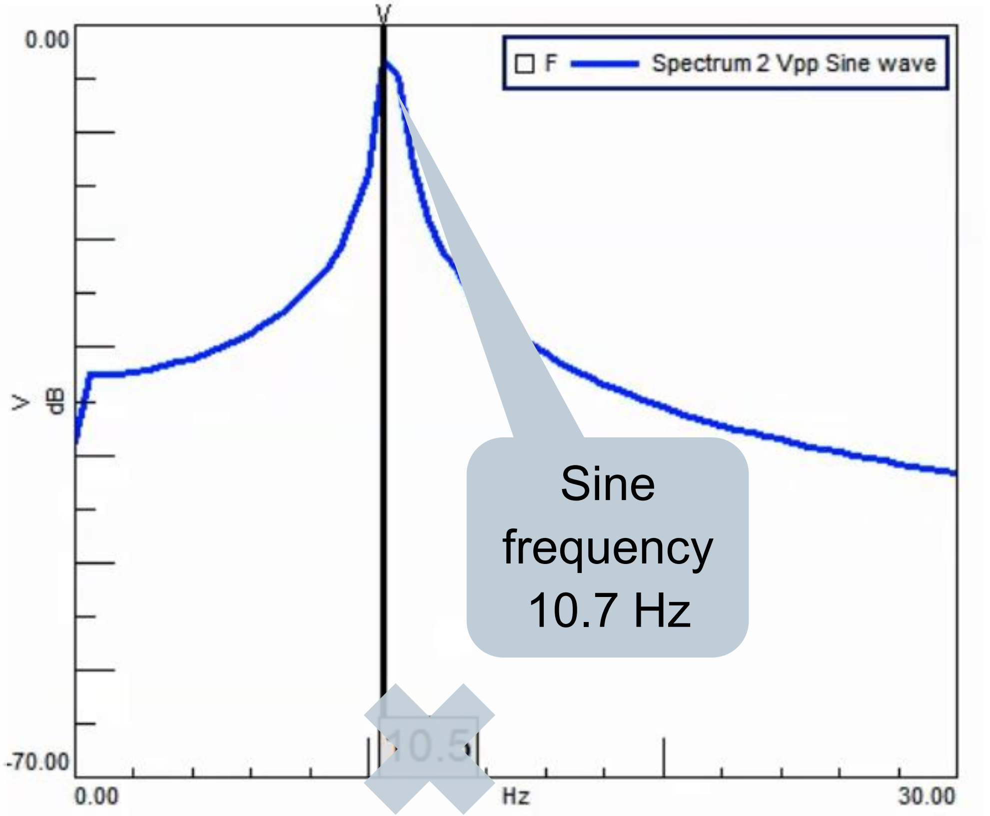
SIEMENS
Ingenuity for life



Slow sweep from 10 to 11 Hz, showing spectrum with 0.5H resolution

Effect of finite observation time

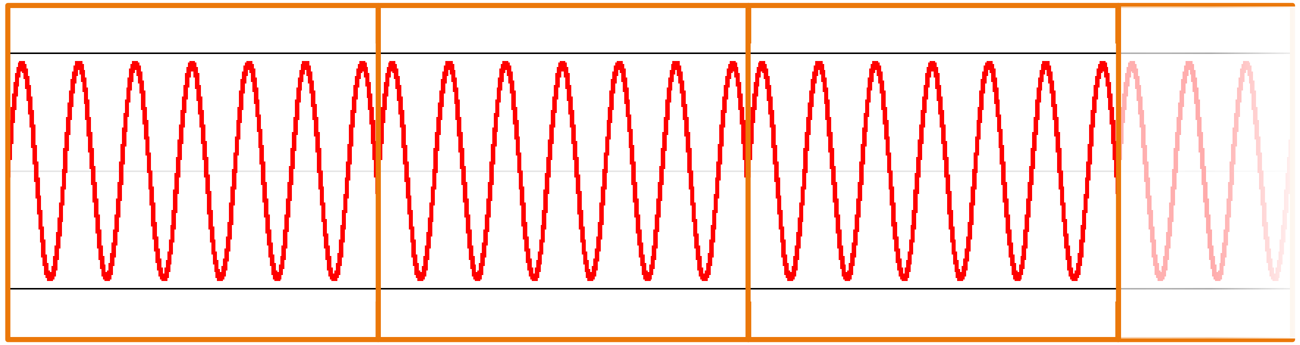


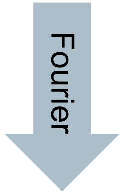


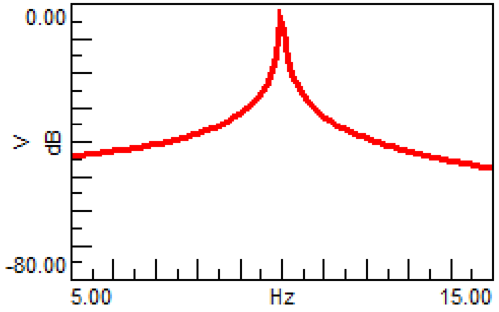
Leakage when signal is not periodic within the observation window

Leakage when sine wave frequency falls between the spectral lines

Why is this happening?
Periodicity assumption



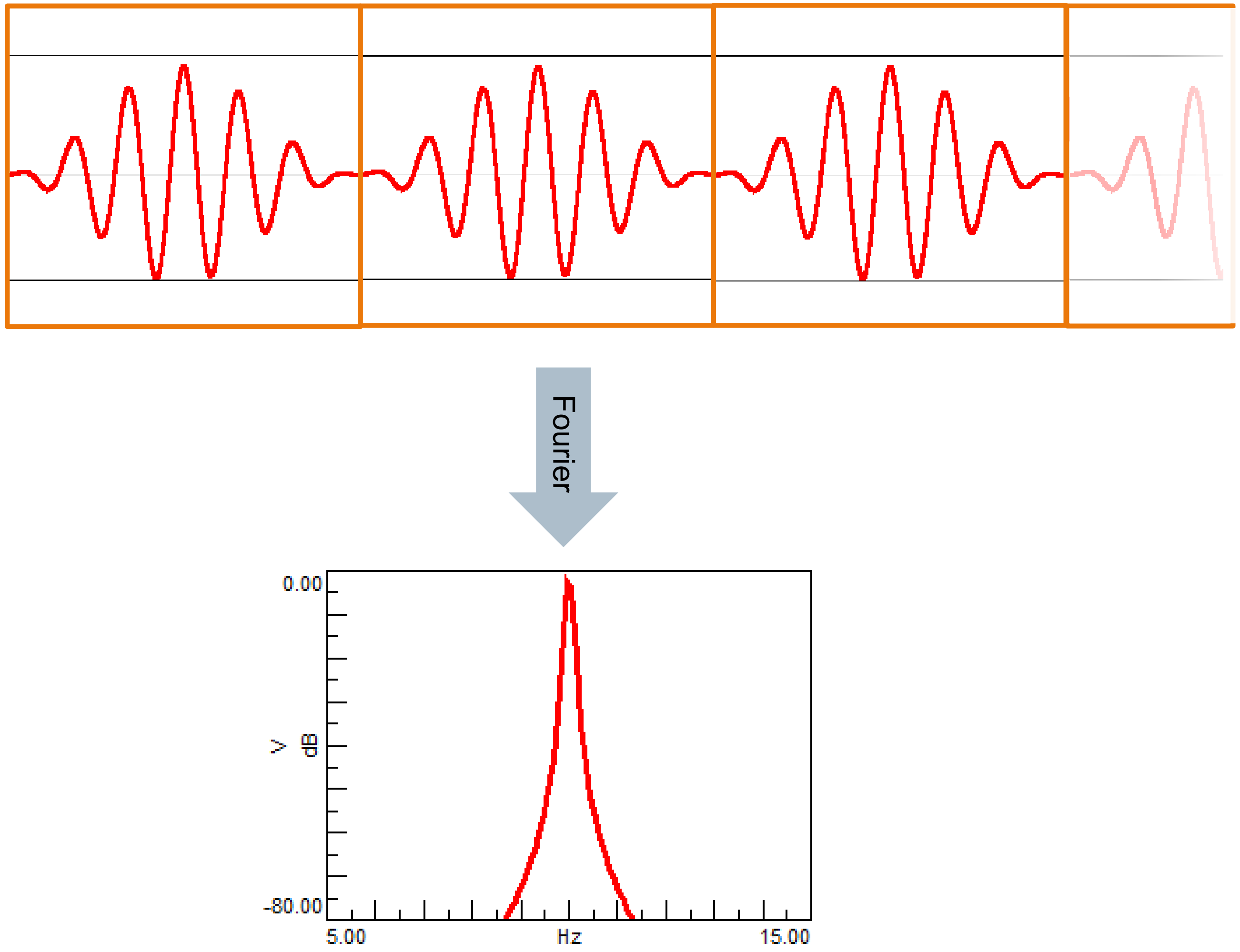




Observation window is assumed to repeat itself, introducing ‘spikes’ in the signal

Reduce the effect of leakage

SIEMENS
Ingenuity for life



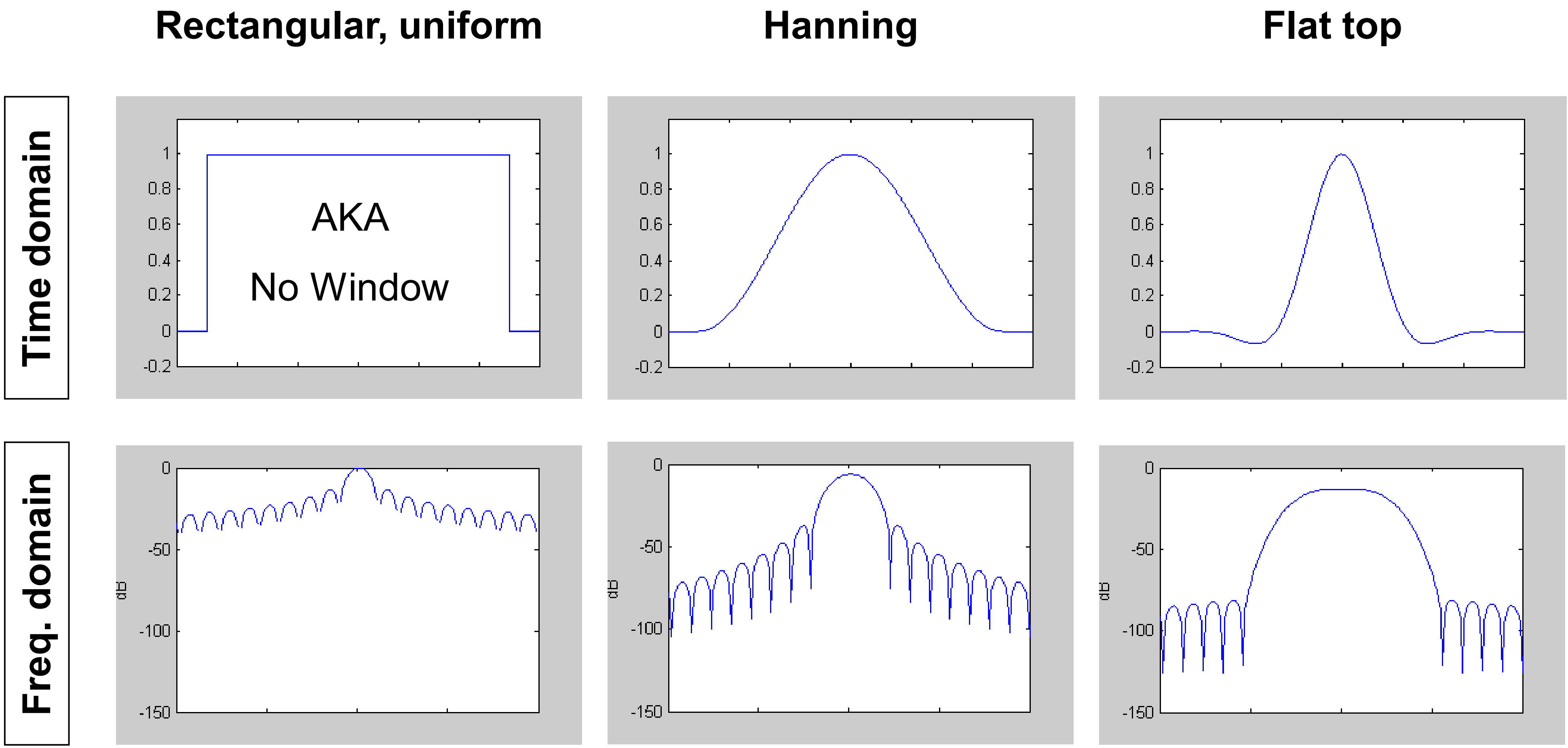
'Force' the input for the Fourier transformation to be periodically expandable

Practical implementation: multiply signal with time domain window to eliminate discontinuities

Effects of time window:

- Improved amplitude estimate
  - → flatten central lobe
- Reduce frequency range of smearing
  - → lower side lobes
- Local smearing of spectral energy due to wider central lobe
  - → lower effective spectral resolution

Window types
most popular ones



Thank you