Fundamentals of DSP
Part 1
Introduction to Digital Signal Processing

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Fundamentals of Digital Signal Processing

Content

Part 1: What is a signal?
- Time and frequency domain
  - Fourier transformation

Part 2: Digitizing signals
- Sampling
- Aliasing

Part 3: Effects to be aware of when converting to digital
- Quantization
- Leakage

Counter-measures to ensure your digital data is valid
Signals

Signal: measurable quantity carrying information about some physical phenomenon
• Pressure, displacement, acceleration, …
• Temperature, voltage, biomedical potential (EKG, EEG, …)

The signal is generated by a structure and detected by a sensor or transducer
• Accelerometer: acceleration → voltage
• Microphone: pressure → voltage
• Strain Gauge: strain (deformation) → voltage
• Thermocouple: temperature changes → voltage

The signal is what you want to analyse in view of a particular problem
Signals
Signal Processing

Signal processing: specific manipulations of the measured signal to

- Extract key information
- Understand physics
- Provide input data for specific analysis
- Confront simulation results with reality
- Modify the signal for specific applications

Signal processing transforms the signal to different domains

- Time domain
- Frequency domain
- Amplitude domain
- Laplace domain
- …
Time, frequency and amplitude domains

Each domain = different coordinate system that is used to view or describe the characteristics of a system or event

- **Time domain**
- **Frequency domain**
- **Amplitude domain**

Each domain highlights a particular aspect of the characteristics of a system or event

• The **time domain** is usually the basis for a description of a system’s dynamic behavior. e.g. differential equation of motion. Events are measured as a function of time.
• The **frequency domain** highlights the periodic characteristics of the system or event.
• The **amplitude domain** represents looks at the probability distribution of the amplitudes.
Fourier transformation
Joseph did help us a lot…

Fourier’s law of heat conduction
\[
\frac{\partial u}{\partial t} = k \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

Analysed in terms of infinite mathematical series

Any signal can be described as a combination of sine waves of different frequencies
Fourier transformation

For mathematicians …

- Convert from time to frequency domain and back
- Fourier integral
- No information is lost when converting!

For engineers …

\[
X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} \, dt
\]

\[
x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} \, d\omega
\]

- Detect sine waves
- Draw line at sine frequency
Some definitions of sine waves

Time domain

- Period: $T_0$ [s]

Frequency domain

- Frequency: $f_0 = 1/T_0$ [Hz]
- Pulsation / circular frequency: $\omega_0 = 2\pi f_0 = 2\pi/T_0$ [rad/s]
Basics of sine waves

Sine Wave Equation

\[ x(t) = A \sin(2\pi ft + \theta) \]

- \( A \) = Amplitude
- \( f \) = Frequency
- \( \theta \) = Phase
- \( t \) = Time

\[ \omega = 2\pi f \]

- \( f \) is in Hz
- \( \omega \) is in radians/sec
Basics of sine waves
Frequency

2 Hz

4 Hz
Basics of sine waves

Amplitude

1 g

0.5 g

Amp

time

1 second

Amp

time

1 second
Basics of sine waves
Amplitude

Scaling can cause amplitude difference!
**Basics of sine waves**

**Phase**

Phase is measured as an angle. The orange signal "lags" the green by about 45° or $\pi/4$ radians. The green signal "leads" the orange by about 315° or $7\pi/4$ radians.

\[ x(t) = A \sin(2\pi ft + \theta) \]
Fourier transformation

Q: What is the output of a Fourier transformation?

A: A spectrum in the frequency domain. It represents a series of sines and cosines in the form of complex numbers. When these numbers are summed, they form the original signal in the time domain.

Solution: \( a + jb = \text{complex number} \)

- \( a = \text{real part} \)
- \( b = \text{imaginary part} \)
- \( j = \sqrt{-1} \)

Note: You will get a complex number for each point in your spectrum.
**Q:** How do I make sense of this complex numbo-jumbo?

**A:** Spectrum most commonly viewed as **Magnitude/Phase**

![Complex Plane Diagram](image)

\[
z = (a^2 + b^2)^{1/2} = [(a + jb)(a - jb)]^{1/2} = \text{magnitude}
\]

\[
a = z \cdot \cos \theta \quad \text{(magnitude \ cos \ phase)}
\]

\[
b = z \cdot \sin \theta \quad \text{(magnitude \ sin \ phase)}
\]

\[
\tan \theta = \frac{z \cdot \sin \theta}{z \cdot \cos \theta} = \frac{b}{a} \quad \theta = \tan^{-1}(b/a)
\]

**AKA:** Bode Plot

![Bode Plot Graph](image)
Selection of domain depends on the application aims
Equivalence of time and frequency domain: no loss of information
PART 2: Digitizing signals
- Sampling
- Aliasing

PART 3: Effects to be aware of when converting to digital
- Quantization
- Leakage

Counter-measures to ensure your digital data is valid
Fundamentals of DSP
Part 2
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Nice theory…
but we need to do this on a computer

Digital Signal Processing: apply manipulations using a computer-based system

- Most transducers output an analog (continuous) signal
- Computers are digital devices (0/1; on/off)
- Convert the sensor signal into a discrete stream of digital information
- Discretization in time and in amplitude
- Massive loss of information when sampling
Sampling analog signals

10 Hz sine wave, sampled at 512 Hz: digital representation looks like a perfect sine
10 Hz sine wave, sampled at 128 Hz: digital representation still looks OK
Sampling analog signals

10 Hz sine wave, sampled at 64 Hz: digital representation starts looking strange…
Sampling analog signals

10 Hz sine wave sampled high (512 Hz, red) and low (64 Hz, green) on a digital oscilloscope
Sampling analog signals
Exploring the limits…

Sampling frequency = sine wave frequency
\[ f_s = f_{\text{sine}} \]

Observed frequency = 0 Hz (DC)

Sampling frequency = 2 x sine wave frequency
\[ f_s = 2 \times f_{\text{sine}} \]

Observed frequency is correct, but it is borderline
(sampling frequency cannot be lowered)
Sampling analog signals
When pushing further, you get aliasing

Sine wave frequency = 20 Hz
Sample rate 21.3 Hz

Observed frequency is wrong: 20 Hz sine wave sampled at 21.3 shows as 1.3 Hz signal
Sampling analog signals
When pushing further, you get aliasing

$$f_{\text{max}} = \frac{f_s}{2}$$

Nyquist frequency
Sampling analog signals
How to prevent aliasing?

Select sample rate to cover full signal bandwidth
If there is no frequency content above Nyquist Frequency, then there is no Aliasing
This is not always practical or possible:
  • Large files sizes
  • Limitations of data acquisition equipment

Limit signal bandwidth using Anti-Aliasing Filter
  • Analog and/or Digital low-pass filters

Low pass filter to prevent aliasing: Removes frequencies that violate Nyquist from analog signal before digitizing

Anti-aliasing filter
Ideal case

Amplitude
0
1

Frequency
0
f_s/2=f_max
Sampling analog signals
How to prevent aliasing?

Make sure the signal does not contain frequencies above half the sample frequency.

Do this by applying a sufficient performing low-pass filter.

Limit signal bandwidth using Anti-Aliasing Filter.

Be aware that the amplitude of the last portion of the spectrum is:

- Analog and/or Digital low-pass filters.

Automatically done in good data acquisition hardware.

Extremely sharp analog filter, but “brick wall” effect is not possible.

Roll off point starts @ 80% of bandwidth.
Aliasing demonstration
Sine sweep from 45 to 82 Hz, sampled at 128 Hz
PART 3: Effects to be aware of when converting to digital

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Counter-measures to ensure your digital data is valid

Thank you
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Digitizing analog signals
Quantization resolution

Sampling in time domain:
Store an amplitude value on a periodic basis (fs)

Amplitude is determined with a discrete resolution

Real values are rounded or truncated to discrete levels

This process causes quantization noise
Digitizing analog signals
Some formulas

- Assume ADC with M bits at the output
- The number of voltage intervals N is given by
  \[ N = 2^M - 1 \]
- The voltage resolution of an ADC is equal to its overall voltage measurement range divided by the number of discrete values:
  \[ \Delta V = \frac{V_{\text{range}}}{2^M} \]
- The Signal-to-quantization-noise ratio is given by
  \[ SQNR = 20 \log_{10}(2^M) \approx 6.02 \times M \text{ dB} \]

<table>
<thead>
<tr>
<th># of bits</th>
<th># of voltage steps</th>
<th>Voltage resolution (for +/- 10V range)</th>
<th>Signal to quantification noise ratio</th>
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<tr>
<td>4</td>
<td>16</td>
<td>1.24 V</td>
<td>24 dB</td>
</tr>
<tr>
<td>8</td>
<td>255</td>
<td>78.1 mV</td>
<td>48 dB</td>
</tr>
<tr>
<td>12</td>
<td>4 095</td>
<td>4.88 mV</td>
<td>72 dB</td>
</tr>
<tr>
<td>16</td>
<td>65 535</td>
<td>0.305 mV</td>
<td>96 dB</td>
</tr>
<tr>
<td>24</td>
<td>16 777 215</td>
<td>1.19 \mu V</td>
<td>144 dB</td>
</tr>
</tbody>
</table>
Quantization demonstration
Quantization demonstration
Quantization demonstration
Mitigating quantization errors

Some terminology

- Headroom or Overhead

Input range

or

Full scale range

or

ADC range
Mitigating quantization errors
Demonstration: 10 Hz, 2 mV sine wave

16 bit ADC, 10 V input range
→ 0.305 mV discretization level

2 mV signal / 0.305 mV = 13
→ truncated to 13 possible levels

→ Large quantization errors!

2 possible solutions:

Use ADC with higher number of bits
Apply gain to the signal prior to ADC
Select the right range, but watch out for overloads!
We don’t have all day…
Finite observation period

Periodic observation: correct amplitude level at correct spectral line
If signal is not periodic in observation window

→ Leakage

Periodic observation: correct amplitude level at correct spectral line

A-periodic observation: up to 63% amplitude error at spectral line closest to correct frequency

Remaining 37% amplitude spread out over entire frequency spectrum

Leakage = severe distortion of spectrum if the signal is not periodic in the observation window
Leakage demonstration

Slow sweep from 10 to 11 Hz, showing spectrum with 0.5H resolution
Effect of finite observation time

Leakage when signal is not periodic within the observation window

Leakage when sine wave frequency falls between the spectral lines

Sine frequency 10.7 Hz
Why is this happening?
Periodicity assumption

Observation window is assumed to repeat itself, introducing ‘spikes’ in the signal
Reduce the effect of leakage

‘Force’ the input for the Fourier transformation to be periodically expandable
Practical implementation: multiply signal with time domain window to eliminate discontinuities

Effects of time window:
- Improved amplitude estimate → flatten central lobe
- Reduce frequency range of smearing → lower side lobes
- Local smearing of spectral energy due to wider central lobe → lower effective spectral resolution
### Window types
#### most popular ones

<table>
<thead>
<tr>
<th>Window Type</th>
<th>Time Domain</th>
<th>Frequency Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular, uniform</td>
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<tr>
<td></td>
<td>AKA No Window</td>
<td></td>
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<tr>
<td>Hanning</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>Flat top</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
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</table>
Thank you